4.1: Systems of Linear Equations

System of equations: a set of equations with common variables.

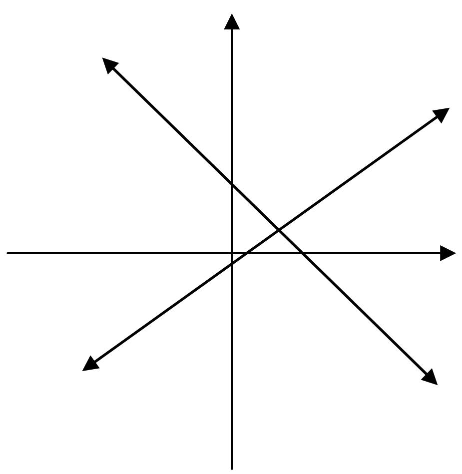
A pair of lines can be given as a system of equations of the form $\begin{cases} ax+by=c \\ dx+ey=f \end{cases}$.

The graph of each equation in the system is a straight line.

Three possibilities for the number of solutions:

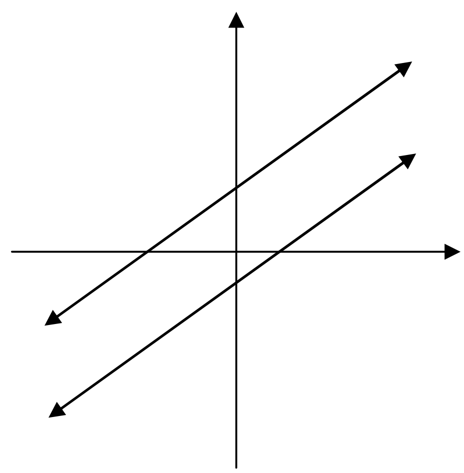
One Solution
(lines intersect at a single point)

Consistent, independent



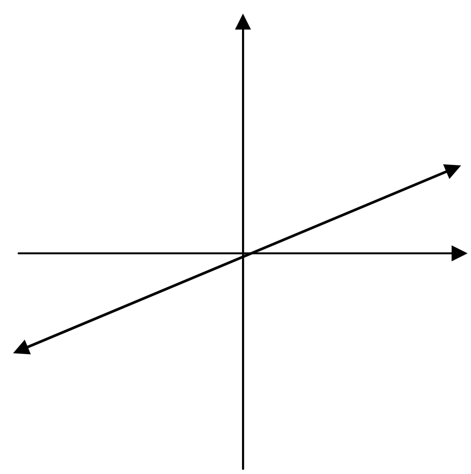
No Solution
(lines are parallel)

Inconsistent



Infinitely Many Solutions
(lines coincide)

Consistent, dependent



In a nutshell:

- "Consistent" means the system can be solved.
- "Inconsistent means it cannot be solved.
- "Independent" means the equations are not multiples of one another.
- "Dependent" means the equations are multiples of one another. They're the same line, just written in a different format.

Methods for solving a system of linear equations:

- Substitution
- Elimination
- Graphing (doesn't give exact solutions)
- Matrices

<u>Example 1:</u> A movie theater sells regular and student tickets. The cost of two regular tickets and one student ticket is $16. The cost of three regular tickets and three student tickets is $30. What is the price of each type of ticket?

Set up the system:

Solve by substitution:

<u>Step 1</u>: Solve one of the equations for one of the variables.
<u>Step 2</u>: "Substitute" the result from Step 1 into the *other* equation.
<u>Notice</u>: You end up with one equation in one variable only.
<u>Step 3</u>: Solve this resulting equation for the other variable.
<u>Step 4</u>: Substitute the result from Step 3 into one of the original equations (it doesn't matter which one). Solve this equation for the remaining variable.
<u>Step 5</u>: Check!

Solve by elimination:

Our goal is to eliminate one of the variables by adding the equations (or multiples of the equations) together.

Step 1: Choose a variable to eliminate. Multiply each equation by an appropriate number. Choose these numbers so that when you add the resulting equations, your chosen variable will go away.
Step 2: Add the resulting equations. This will give you one equation in one variable.
Step 3: Solve this new equation for its variable.
Step 4: Substitute the result from Step 3 into one of the original equations (it doesn't matter which one). Solve this equation for the remaining variable.
Step 5: Check!

The above example had a unique solution. What about cases where there is no solution or an infinite number of solutions?

If there's no solution (inconsistent): You'll get a contradiction (for example, $0 = 17$).

Example 2: Solve the system $\begin{cases} 3y - 2x = 15 \\ 4x - 6y = -6 \end{cases}$.

If there are infinitely many solutions (consistent, dependent) (same line): You'll get a statement that's always true (usually $0 = 0$). In this case your solution will have one variable written in terms of the other.

Example 3: Solve the system $\begin{cases} 2x + 4y = 6 \\ 3x + 6y = 9 \end{cases}$.

Example 4: A pet supply company makes two types of fancy dog beds: the *Couch* and the *Throne*. Each *Couch* requires 30 minutes in the cutting department and 45 minutes in the sewing department. Each *Throne* requires 45 minutes in cutting and 75 in sewing. If the company has allocated a total of 80 hours of cutting time and 130 hours of sewing time to dog beds, how many of each type can be made?

Example 5: An apparel shop sells shirts for $40 and ties for $15. Its entire stock is worth $14,165, but sales are slow and only half the shirts and one-third of the ties are sold, for a total of $6295. How many shirts and ties are left in the store?

Example 6: For a certain area, when the supply of available square bales of hay was at 30,000 bales and the demand was for 26,000 bales, the price per bale was \$8.50. When the supply was 25,000 bales and the demand was 23,000 bales, the price per bale rose to \$10.75. Assuming that the price-supply and price-demand equations are linear, calculate:

a) the price-supply equation.
b) the price-demand equation.
c) the equilibrium price and quantity.

Example 7: After enjoying detailing cars for his friends and family for years, Frodo has decided to open his own car detailing shop. To keep his shop going, he must pay fixed costs of \$1300 per month (lease of the space, basic utilities, etc.) even if he doesn't clean any cars. He charges an average of \$200 for each detail job. Each detail job costs Frodo approximately \$20 in supplies, electricity, and water; each job also costs Frodo \$120 in terms of time. (He made \$30/hour at his old job, and he figures 4 hours to do each car.)

a) Find the cost equation and revenue equation for Frodo's business.
b) How many cars must Frodo clean each month for his business to break even?