8.1: Sample Spaces, Events, and Probability

An *experiment* is an activity with observable results. An experiment that does not always give the same result, even under the same conditions, is called a *random experiment*. Repetitions of an experiment are called *trials*.

> Examples: rolling dice, drawing cards, counting the number of defective radios in a shipment, etc.

In probability theory, we'll usually use the word *experiment* to mean a *random experiment*.

For a given experiment, we can make a list of outcomes of the experiment, called *simple events*, such that in each trial, one and only one of the simple events will occur. The set of all such simple events is called the *sample space*.

Any subset of the sample space is called an *event*. If such a subset contains more than one element of the sample space, we call the subset a *compound event*. If an event $E = \varnothing$, we call the event E an *impossible event*. If S is the sample space and $E = S$, then E is a *certain event*.

Example 1: We roll a single six-sided die.

Example 2: We spin a roulette wheel (a wheel that has the integers 0 to 36 on it).

Example 3: The manager of a local cinema records the number of patrons attending a particular movie. The theater has a seating capacity of 500.

a. What is an appropriate sample space for this experiment?

b. Describe the event E that fewer than 50 people attend the movie?

c. Describe the event F that the theater is more than half full at the movie?

Example 4: An experiment consists of studying the composition of a three-child family.

a. Describe an appropriate sample space for this experiment if we are interested in the genders of the children in the order of their births.
b. Describe the event E that there are two girls and a boy in the family.
c. Describe the event F that the oldest child is a girl.
d. Describe the event G that the oldest child is a girl and the youngest is a boy.
e. Describe an appropriate sample space if we are only interested in the number of girls and not the order.

Example 5: Roll a pair of dice. What is the most useful sample space? Describe the following events:

a. The sum is 6.
b. The sum is 2.
c. The sum is more than 9.

Probability:

In many sample spaces, all the outcomes, or simple events, are equally likely to occur. In these cases, we use the *equally likely assumption*. If several choices are possible for an experiment's sample space, it is often best to choose one in which all outcomes are equally likely.

Equally likely assumption:

If all events in a sample space are equally likely to occur, the probability of each is $\frac{1}{n}$, where n is the number of simple events in the sample space.

The equally likely assumption results in a basic principle of probability.

Basic Probability Principle

Let S be a sample space of equally likely outcomes, and let the event E be a subset of S. Then the probability that event E occurs is

$$P(E) = \frac{n(E)}{n(S)}.$$

Example 6: Flip two fair coins. What is the probability of getting

a. Two tails?
b. At least one head?
c. Exactly one head?
d. Two heads or two tails?
e. Three tails?

> Important Note: For any event E,
>
> $$0 \le P(E) \le 1.$$
>
> If $P(E) = 0$, then E is an *impossible event*.
> If $P(E) = 1$, then E is an *certain event*.

Example 7: Draw a single card from a standard card deck. What is the probability that it is

a. A spade?
b. A queen?
c. Black?
d. Higher than a 7? (assume aces high.)

Example 8: Roll a single die. What is the probability of rolling

a) a 5?
b) a prime number?
c) a multiple of 3?
d) a number larger than 10?
e) a number smaller than 10?

Example 9: Roll two dice. What is the probability that the sum is 7? That the sum is less than 6?

Example 10: Jennifer's electricity went out last night. Not having a flashlight, she had to choose a T-shirt at random in the dark. She owns 10 white T-shirts, 5 gray ones, 6 black ones and 2 red ones. What is the probability that she chose a gray T-shirt?

Example 11: Suppose that Joe, Steve, Suzy, and Lisa work for the same company. The company wants to send two representatives to a particular conference and needs two to stay home and take care of the customers. All of them want to attend the conference, so they decide to put their names in a hat and draw two at random. What is the probability that Suzy and Lisa are selected?

Example 12: The tuition categories for North Harris College students in the spring semester of 2007 are given in the table below.

In-District	9,794
Out-of-District	428
Out-of-State	104
International	250
Other	9

a) What is the probability that a randomly selected student lives in-district?

b) What is the probability that a randomly selected student is an international student?

Example 13: The age distribution of Spring 2007 North Harris College students in the spring semester of 2007 is given in the table below.

Age of Students	
Under 20	33.4%
20-24	35.6%
25-29	11.3%
30-39	11.5%
40-49	6.3%
50 and Over	2.0%
Total	100%

What is the probability that a randomly selected student is

a) under 20 years old?

b) Over 40?

c) Over 60?

Probability assignments:

Suppose $S = \{e_1, e_2, \ldots e_n\}$ is a sample space. It contains *n* simple events, or outcomes. To each outcome, we can assign a number $P(e_i)$, called the *probability of the event* e_i.

Rules for probabilities:

- The probability of a simple event is a number between 0 and 1, inclusive. In other words, $0 \le P(e_i) \le 1$.
- The probabilities of all the simple events in the sample space add up to 1. In other words, $P(e_1) + P(e_2) + \ldots + P(e_n) = 1$.

Any probability assignment that meets these conditions is called an *acceptable probability assignment.*

A probability assignment that reflects the actual or expected percentage of times a simple event occurs is called *reasonable*. In other words, it is reasonable if it makes sense based on the real world.

Example 14: Roll a single die.

Probability of an event E:

Given a sample space *S* and an acceptable probability assignment, then for any event *E* ($E \subset S$), the following rules apply:

- If $E = \varnothing$, then $P(E) = 0$.
- If *E* is a simple event, then $P(E)$ is given by the original probability assignment.
- If *E* is a compound event, then $P(E)$ is the sum of the probabilities of all the simple events in *E*.
- If $E = S$, then $P(E) = P(S) = 1$.

Steps for finding $P(E)$:

1. Set up an appropriate sample space.
2. Assign acceptable probabilities to each simple event.
3. $P(E)$ is the sum of the probabilities of all the simple events in *E*.

There are two basic methods of assigning probabilities.

- Theoretical: no experiments are conducted—use assumptions and reasoning.
- Empirical: use results of actual experiments. (This is also called *relative frequency probability*.)

Empirical (relative frequency) probability approximation:

$$P(E) \approx \frac{\text{frequency of occurence of } E}{\text{total number of trials}} = \frac{f(E)}{n}$$

(The larger n is, the better the approximation.)

Example 15: Suppose that we survey 134 students at Lone Star College-North Harris and find that 97 of them own an iPhone. Use this information to estimate the probability that a randomly selected LSC-NH student owns an iPhone. What is the probability that a randomly selected student does not own an iPhone?

Example 16: Draw 5 cards from a standard 52-card deck.

a. What is the probability of getting 5 spades?
b. What is the probability of getting 2 kings, 2 queens, and a jack?

Example 17: Joe and Susan Thomas work in a department with 12 other people. Four employees are to be chosen to attend an important conference in the Bahamas. What is the probability that both Joe and Susan will be chosen?

Example 18: Suppose that you had written six thank-you cards and addressed six envelopes. While you were away from your desk, your young child, attempting to be helpful, put the cards into the envelopes randomly, stamped and mailed them.

a) What is the probability that all the cards were put into the correct envelopes?
b) What is the probability that only two cards were sent to the wrong people?

Example 19: Suppose that a shipment of 100 radios contains three that are defective. If a quality-control engineer selects a random sample of 5 radios, what is the probability that the sample contains at least one defective radio?

Example 20: My tack locker has a combination lock with a wheel that must be turned to 3 numbers in succession. The wheel contains the numbers 0 through 39, and such locks do not involve repeated numbers. What is the probability of guessing the correct combination?

Example 21: In the Lotto Texas game, players choose six numbers from the numbers 1-54, without repeating numbers. The player wins the big jackpot if he chooses all six winning numbers, regardless of order. He can win a smaller 2^{nd}, 3^{rd} or 4^{th} prize by matching 5, 4, or 3 of the six winning numbers.

What is the probability he wins the jackpot?

What is the probability he wins 2^{nd} prize?

3^{rd} prize?

4^{th} prize?