

3.4: Present Value of an Annuity; Amortization

Present Value of an Ordinary Annuity

$$PV = PMT \left[\frac{1 - (1 + i)^{-n}}{i} \right]$$

where

PMT = periodic payment (made at end of period)

$i = \frac{r}{m}$ = rate per period

n = number of payments (periods)

PV = present value of all payments

Example 1: How much should you deposit into an account that pays 6% compounded semiannually so that \$1,000 may be withdrawn every 6 months for three years?

The *amortization* of a debt is the process of paying it off in equal installments. For example, if I buy a new car and don't have the cash for it, I *amortize* the debt by making equal monthly payments.

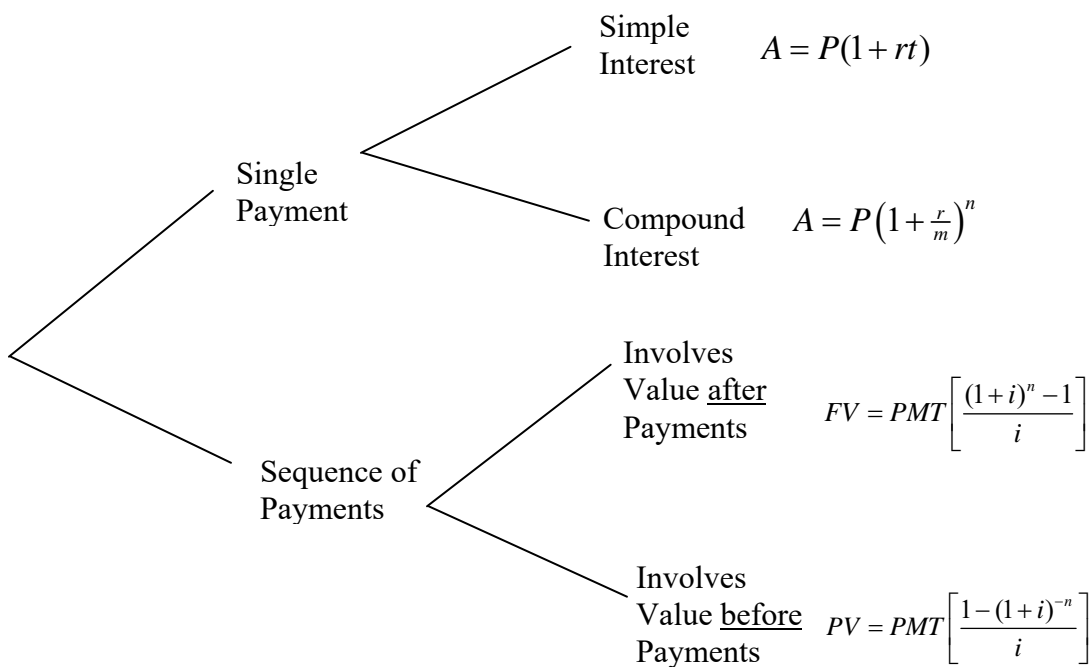
Example 2: Suppose you want to finance an \$800 television. The electronics store is willing to finance it for 18 months at 18% compounded monthly.

- a. What are the monthly payments?
- b. How much total interest will you pay?

Example 3: I buy a car for \$20,000. I put \$800 down and the dealer gives me \$1800 for my trade-in. I finance the rest at 5.5% for five years (compounded monthly). What are my monthly payments? How much total money do I pay for the car? How much interest?

Example 4: Scott and Jennifer are considering buying a house. The house they like costs \$110,000, and they have saved \$10,000 for a down payment.

- a. What will be their monthly payment for a 30-year loan at 5% (compounded monthly)? How much interest will they pay?
- b. What will be their monthly payment for a 15-year loan at 5% (compounded monthly)? How much interest will they pay?
- c. What will be their monthly payment for a 15-year loan at 4.6% (compounded monthly)? How much interest will they pay?

Summary of formulas:**Amortization schedules:**

Example 5: Suppose that Scott and Jennifer decided to buy the \$110,000 house with the \$10,000 down payment, financed for 30 years at 5%. How much of their first 3 payments went to interest? How much would have gone toward interest had they opted for the 15-year loan at 5%?

Example 6: Scott and Jennifer ended buying a \$135,000 house. They chose the 30-year mortgage at 6% annual interest rate, with a 3% down payment. Six years later, they were able to refinance at 3.875% with no closing costs. This time, they chose a 15-year mortgage.

- a. What was their remaining balance when they refinanced?
- b. How much interest did they pay during the first six years (on the original mortgage)?
- c. How much interest would they have paid during the remaining 24 years of the original mortgage?
- d. What are the payments on the new mortgage?
- e. Suppose that after paying on the new mortgage for nine years (with no additional principal payments), the house is worth \$200,000. How much equity do they have?