4.4: Matrices-Basic Operations

We will learn how to add matrices and how to multiply a matrix by a number (scalar).

Equality:

Two matrices are *equal* if they are the same size *and* all the corresponding elements are equal.

Example 1:

$$A=\begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix},\ B=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix},\ C=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix},$$

$$D=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix},\ E=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix},\ F=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

Addition:

Matrices can be added only if they are the same size. In this case, we add the corresponding elements in the matrices.

Example 2:

$$\begin{bmatrix}1 & 3\\ 5 & 7\end{bmatrix}+\begin{bmatrix}1\\ 7\end{bmatrix}=$$

$$\begin{bmatrix}2 & 4\\ 4 & 2\end{bmatrix}+\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}=$$

$$\begin{bmatrix}-1\\ 2\\ 3\end{bmatrix}+\begin{bmatrix}4\\ -5\\ 6\end{bmatrix}=$$

Addition of numbers is associative and commutative. Addition of matrices is associative and commutative also.

- <u>Commutative</u>: $A+B=B+A$
- <u>Associative</u>: $(A+B)+C=A+(B+C)$

A *zero matrix* is a matrix with zero in all positions. The following are zero matrices of different sizes:

The *negative of a matrix A*, denoted $-A$, is the matrix with all elements that are the opposites of the corresponding elements in the matrix *A*.

Example 3:

Subtraction:

As with addition, subtraction can be performed only if matrices are the same size. The difference $A-B$ is defined to be $A+(-B)$. So to subtract, we just subtract the corresponding elements.

Example 4:

$$\begin{bmatrix} 1 & 2 & -6 \\ -3 & 4 & 5 \end{bmatrix} - \begin{bmatrix} 0 & -2 & -2 \\ 4 & 6 & 5 \end{bmatrix} =$$

Multiplication of a matrix by a number:

The product of a number *k* and a matrix *M*, denoted by *kM*, is the matrix formed by multiplying each element of *M* by *k*. This is often called *scalar multiplication*.

Example 5:

$$4\begin{bmatrix} 1 & 3 \\ -2 & 7 \\ 0 & -4 \end{bmatrix} =$$

Product of a row matrix and a column matrix (in that order):

The product of a $1 \times n$ row matrix A and an $n \times 1$ column matrix is the 1×1 matrix given by

$$AB = \begin{bmatrix} a_1 & a_2 \cdots a_n \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} a_1 b_1 + a_2 b_2 + \cdots + a_n b_n \end{bmatrix}.$$

<u>Note</u>: For this formula to hold, they must be in this order: row $\times$ column. If they are in the other order (column $\times$ row), you get a different result. We'll see one like this later.

<u>Note</u>: The number of elements in the row and column must be the same in order for the multiplication to be defined.

<u>**Example 6:**</u>

$$\begin{bmatrix} 1 & -2 & 3 & -5 \end{bmatrix} \begin{bmatrix} -4 \\ 0 \\ 2 \\ -4 \end{bmatrix} =$$

$$\begin{bmatrix} 21 & -32 & 19 \end{bmatrix} \begin{bmatrix} 11 \\ 23 \\ 13 \end{bmatrix} =$$

$$\begin{bmatrix} 3 & -2 & 5 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} =$$

Matrix multiplication:

If A is an $m \times p$ and B is a $p \times n$ matrix, then the product of these is denoted AB and it is an $m \times n$ matrix.

The entries in the matrix AB are formed as follows: the element in the ith row and jth column is the product of the ith row of A with the jth column of B.

<u>*Important Note*</u>: If the number of columns of A is not equal to the number of rows of B, the product AB is not defined! The matrices cannot be multiplied!!

Example 7:

$$A=\begin{bmatrix}2 & 1 & 3\\ 0 & -2 & -1\end{bmatrix},\ B=\begin{bmatrix}1 & 0\\ 4 & -2\\ 1 & 1\end{bmatrix}$$

$$AB=\begin{bmatrix}2 & 1 & 3\\ 0 & -2 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ 4 & -2\\ 1 & 1\end{bmatrix}=$$

$$AB=\begin{bmatrix}2 & 1 & 3\\ 0 & -2 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ 4 & -2\\ 1 & 1\end{bmatrix}=$$

$$AB=\begin{bmatrix}2 & 1 & 3\\ 0 & -2 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ 4 & -2\\ 1 & 1\end{bmatrix}=$$

$$AB=\begin{bmatrix}2 & 1 & 3\\ 0 & -2 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ 4 & -2\\ 1 & 1\end{bmatrix}=$$

Example 8:

$$\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}\begin{bmatrix}3 & 2\\ -1 & 4\end{bmatrix}=$$

Example 9:

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} =$$

Example 10:

$$\begin{bmatrix} -1 & 3 \\ 4 & 2 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 6 & 1 & -2 \end{bmatrix} =$$

Example 11:

$$\begin{bmatrix} 1 & 3 & 1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} =$$

Example 12:

$$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ -1 & -3 \end{bmatrix} =$$

Example 13:

$$\begin{bmatrix} 2 & 6 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} =$$

Example 14:

$$\begin{bmatrix} 2 & -4 & 3 \\ -3 & 1 & 5 \\ 10 & -2 & 7 \end{bmatrix}^2$$