5.2: Systems of Linear Inequalities in Two Variables

Solving systems of linear inequalities graphically:

Now we'll solve systems of several inequalities.

We want to find the graph of all ordered pairs (x, y) that simultaneously satisfy all the inequalities in the system. The graph is called the *solution region*, or the *feasible region*, for the system. To find the solution region, we graph each inequality in the system (this will give a shaded area for each). The area that is included in *all* of them is the feasible region.

A *corner point* of a feasible region is a point in the solution region that is the intersection of two boundary lines.

Example 1: Solve the following system and find the corner points.

$$x - 2y < 6$$
$$2x + y \geq 4$$

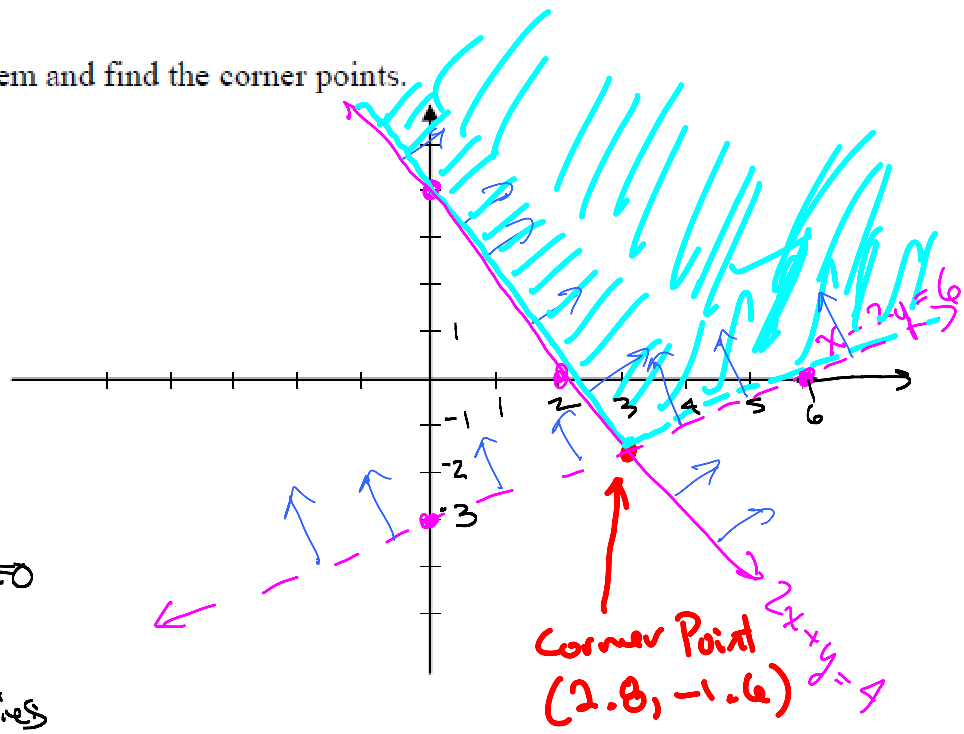
Lines:

$x - 2y = 6 \quad (0,-3)(6,0)$

$2x + y = 4 \quad (0,4)(2,0)$

Recall
To find x-intercept, set $y=0$ and solve for x

To find y-intercept, set $x=0$ and solve for y.



Test points in the inequalities

$x - 2y < 6$

Test pt $(0,0)$: $0 - 2(0) < 6$

$0 < 6$ True

shade half contains $(0,0)$

$2x + y \geq 4$

Test $(0,0) \Rightarrow 0 \geq 4$ False

shade half not containing $(0,0)$

Find corner point:

$x - 2y = 6 \xrightarrow{(-2)} -2x + 4y = -12$

$2x + y = 4 \longrightarrow 2x + y = 4$

$5y = -8 \Rightarrow y = -\frac{8}{5} = -1.6$

$\begin{cases} x - 2y = 6 \\ 2x + y = 4 \end{cases}$ $\begin{matrix} \longrightarrow x - 2y = 6 \\ \xrightarrow{(2)} 4x + 2y = 8 \end{matrix}$

$5x = 14$

$x = \frac{14}{5} = 2\frac{4}{5} = 2.8$

$\boxed{\text{Corner Point: } (-1.6, 2.8)}$

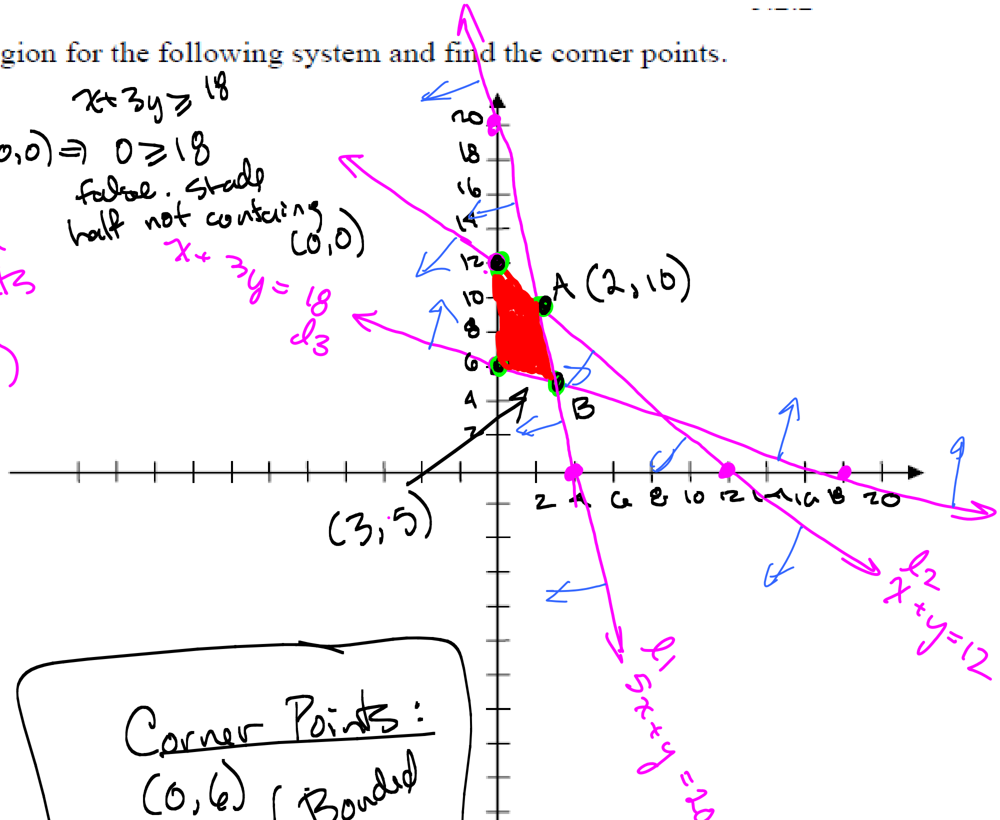
Example 2: Graph the feasible region for the following system and find the corner points.

$$5x + y \le 20$$
$$x + y \le 12$$
$$x + 3y \ge 18$$
$$\left.\begin{aligned} x &\ge 0 \\ y &\ge 0 \end{aligned}\right\}$$ nonnegative constraints (Put us in Quadrant)

$x + 3y \geq 18$

$(0,0) \Rightarrow 0 \geq 18$

false. Shade half not containing $(0,0)$



l_1: $5x + y = 20$

$(0, 20)$ $(4, 0)$

l_2: $x + y = 12$

$(0, 12)$ $(12, 0)$

l_3: $x + 3y = 18$

$(0, 6)$ $(18, 0)$

Corner Points:

$(0,6)$

$(0,12)$

$(3,5)$

$(2,10)$

(Bounded Region)

$5x + y \leq 20$

Test point: $(0,0) \Rightarrow 5(0) + 0 \leq 20$

$0 + 0 \leq 20$

true! $\rightarrow 0 \leq 20$

Shade half-plain containing $(0,0)$

$x + y \leq 12$

$(0,0) \Rightarrow 0 \leq 12$

true

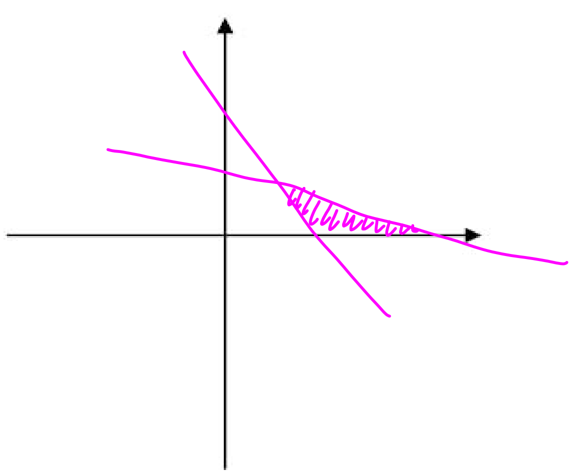
Find corner points:

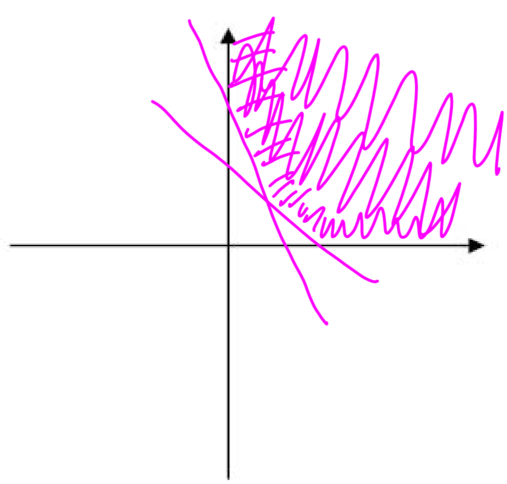
$(0,6)$, $(0,12)$ } by inspection

Find corner Point A

$x + y = 12$

Bounded and unbounded solution regions:





Bounded Region: can be enclosed in a circle

Unbounded region: cannot be enclosed in a circle

Example 3: Budget Cat Food Company supplies two distributors. One needs at least 200 boxes of cat food monthly, and the other needs at least 400. Budget Cat Food can make 800 boxes at the most. Write a system of inequalities to describe the situation and then graph the feasible region.

Distributor A needs 200^+ boxes

Distributor B needs 400 boxes

Let x = number of boxes of cat food for Distributor A

Let y = number of boxes for Distributor B

System of inequalities

$x \geq 200$

$y \geq 400$

$x + y \leq 800$ → (0, 800) (800, 0)

$x \geq 0$

$y \geq 0$

Corner Pts:

A (200, 400)

B (200, 600)

C (400, 400)