6.3: The Dual Problem: Minimization with $\geq$ Problem Constraints

The simplex method can be modified to solve minimization problems.

The transpose of a matrix:

The transpose of a matrix A is called A^T and is formed by interchanging the rows and columns of A.

Example 1: Find the transpose of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$.

The dual problem:

Every minimization problem with $\geq$ constraints can be associated with a maximization problem with $\leq$ constraints. This maximization problem is called the *dual problem*.

Example 1: Minimize $C = 2y_1 + y_2$

Subject to
$$\begin{aligned} y_1 + y_2 &\geq 8 \\ y_1 + 2y_2 &\geq 4 \\ y_1 &\geq 0 \\ y_2 &\geq 0 \end{aligned}$$

First, we create a matrix A using the constraints and the objective function, with the objective function on the bottom row:

Next, form the transpose A^T :

From the transpose, write a new linear programming problem with new variables:

The dual problem is:

<u>Theorem of Duality</u>:

The objective function *w* of a minimizing linear programming problem takes on a minimum value if and only if the objective function *z* of the corresponding dual maximizing problem takes on a maximum value. The maximum value of *z* is equal to the minimum value of *w*.

So, after forming the dual problem, use the simplex method to solve it.

- For slack variables, use the variables of the original minimization problem.
- When writing the solution, the values of the original variables are read from the bottom row.

Example 2: (Example from Section 5.3—plant food)

Minimize $C = 30x_1 + 35x_2$

Subject to
$$\begin{aligned} 20x_1 + 10x_2 &\geq 460 \\ 30x_1 + 30x_2 &\geq 960 \\ 5x_1 + 10x_2 &\geq 220 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \end{aligned}$$

Example 3: An oil company operates two refineries in a certain city. Refinery I has an output of 200, 100, and 100 barrels of low-, medium-, and high-grade oil per day, respectively. Refinery II has an output of 100, 200, and 600 barrels of low-, medium-, and high-grade oil per day, respectively. The company wishes to produce at least 1000, 1400, and 3000 barrels of low-, medium-, and high-grade oil to fill an order. If it costs \$20,000/day to operate refinery I and \$30,000/day to operate refinery II, determine how many days each refinery should be operated to meet the requirements of the order at minimum cost to the company.