

7.2: Sets

Definition: A *set* is a well-defined collection of objects. Each object in a set is called an *element* of that set.

Examples of sets:

Not sets:

Sets can be finite or infinite.

Examples of finite sets:

Examples of infinite sets:

Notation:

- We usually use capital letters for sets.
We usually use lower-case letters for elements of a set.
- $a \in A$ means a is an element of the set A .
 $a \notin A$ means a is not an element of the set A .
- The *empty set* is the set with no elements. It is denoted \emptyset . This is sometimes called the *null set*.
- $S = \{x \mid P(x)\}$ means “ S is the set of all x such that $P(x)$ is true”. (called rule notation or set roster notation).

Example: $S = \{x \mid x \text{ is an even positive integer}\}$ means $S = \{2, 4, 6, 8, \dots\}$

Definition: We say two sets are *equal* if they have exactly the same elements.

Subsets:

Definition: If each element of a set A is also an element of set B , we say that A is a *subset* of B . This is denoted $A \subseteq B$ or $A \subset B$. If A is not a subset of B , we write $A \not\subseteq B$.

Definition: We say A is a *proper subset* of B if $A \subseteq B$ but $A \neq B$. (In other words, every element of A is also an element of B , but B contains at least one element that is not in A .)

Note on notation: Some books use the symbol \subset to indicate a proper subset. Some books use \subseteq to indicate any subset, proper or not.

Definition: The set of all elements under consideration is called the *universal set*, usually denoted U .

Example: If you're dealing with sets of real numbers, then U is the set of all real numbers. So "Wednesday" would not be an element of U , but 5.7 would be in U .

Example 1: Consider these sets.

$$A = \{1, 2, 3, 4, 5, 6\}$$

$$B = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$C = \{1, 3, 5, 2, 4, 6\}$$

Note:

- \emptyset is a subset of every set. (i.e. $\emptyset \subseteq A$ for every set A .)
- Every set is a subset of itself. (i.e. $A \subseteq A$ for every set A .)

Example 2: List all subsets of $\{1, 2, 3\}$.

Note: If a set has n elements, how many subsets does it have?

Set operations:

- Union \cup : $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- Intersection \cap : $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- Complement A' or A^C or A^{\sim} : $A' = \{x \in U \mid x \notin A\}$.

Note: $A \subseteq (A \cup B)$ and $B \subseteq (A \cup B)$.

$(A \cap B) \subseteq A$ and $(A \cap B) \subseteq B$.

Definition: We say that A and B are *disjoint sets* if $A \cap B = \emptyset$.

Example 3: $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$$H = \{1, 3, 5, 7\}$$

$$K = \{1, 3, 5, 6\}$$

$$J = \{2, 4, 6, 8\}$$

$$L = \{2, 3, 4\}$$

Venn Diagrams: These help us visualize set relationships and operations.

Example 4: Draw Venn diagrams for $A \cup B$, $A \cap B$, A' , $(A \cap B)'$, $A' \cap B'$, and $A' \cap B$.

Example 5: On a Venn diagram, shade $A \cup B \cup C$, $A \cap B \cap C$, and $(A \cup B') \cap C$.

Example 6: Consider a group of students. 30 of them are enrolled in a math course and 35 are enrolled in an English course. 13 of the students are enrolled in an English course and also a math course. How many students are enrolled in math or English?

Notation: $n(A)$ means the number of elements in set A .

Addition principle for Counting

For any two sets A and B ,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

If A and B are disjoint ($A \cap B = \emptyset$), then $n(A \cup B) = n(A) + n(B)$.

Example 7: 100 students are surveyed to determine if they had watched ESPN or Fox Sports Channel in the last 3 months. The results show that 65 students watched ESPN, 55 watched Fox Sports, and 30 watched neither.

- How many people watched ESPN but not Fox Sports?
- How many people watched Fox Sports but not ESPN?
- How many watched both networks?

Example 8: I want to buy a car from Jay Austin Motors. Of all the cars on the lot, 89 cars have navigation systems, 100 have touch-screen controls, and 74 have blind spot alert systems. 32 cars have both navigation systems and blind spot alert, 40 have both a touch screen and a blind spot alert system, and 53 have a touch screen and a navigation system. Twelve cars have all three features, and 21 cars are base models with none of these features.

I strongly dislike having a touch screen, but I would like a navigation system and a blind spot alert system. How many cars do I have to choose from?

How many cars are on Jay Austin's lot?