7.3: Basic Counting Principles

Multiplication principle for counting:

This principle is used to analyze sets which are determined by a sequence of operations.

Example 1: A company sells football jerseys. The jerseys come in sizes S, M, L, XL, and XXL. They also come in two colors: red for home games and white for away games. How many total types of jerseys does the company make?

Multiplication Principle

Suppose that n choices must be made, with

m_1 ways to make choice 1,
m_2 ways to make choice 2,
m_3 ways to make choice 3,

.
.
.

m_n ways to make choice n.

Then there are $m_1 \cdot m_2 \cdot \ldots \cdot m_n$ ways to make the entire sequence of choices.

Example 2:   How many license plate "numbers" can be formed by using a letter, followed by two digits, followed by three more letters?

How many can be formed assuming adjacent letters and numbers must be different?

How many can be formed assuming letters and numbers cannot be repeated?

Products like this occur so frequently that special counting formulas and notations have been developed for them. These formulas use a function called the factorial.

The Factorial:

For a natural number (positive integer) n, $n!$ is called "n-factorial". It is defined as follows:

$$n! = n(n-1)(n-2)\ldots(3)(2)(1)$$
$$n! = n(n-1)!$$
$$0! = 1$$

Example 3:

$6! =$

$\dfrac{8!}{7!} =$

$\dfrac{97!}{3!94!} =$

Note: Factorials grow very rapidly!

Example 4:   Compare 5!, 10!, and 15!.