<u>8.5: Random Variable, Probability Distribution, and Expected Value</u>

A *random variable* is a variable that takes on numerical values representing the outcomes of a probability experiment. Each simple event in a sample space *S* is associated with a value of the random variable.

A *probability distribution* is a function that assigns a probability to each value of the random variable. If there are a finite number of outcomes, the sum of all their probabilities must equal 1. Each probability must be between 0 and 1, inclusive.

<u>Notation</u>: We usually use capital letters to denote a random variable, and use lower-case letters to denote the values taken on by the random variable.

The probability of random variable X taking on the value x is denoted as $P\left(X = x\right)$ or as $p(x)$.

A probability distribution can be represented graphically as a histogram.

<u>Example 1:</u> Write the probability distribution associated with the rolling of a single fair die. Sketch the histogram.

Expected value of a random variable:

Suppose a probability experiment is repeated a large number of times. We can record the value of the random variable produced by each trial (repetition) of the experiment. If we calculate the mean (average) of all these values, that mean should be close to the *expected value* of the random variable. As the number of trials increases, the mean of the outcome variable should get closer and closer to the expected value.

<u>Expected value</u>**:**

Suppose that a random variable x can take on the n values $x_1, x_2, ..., x_n$. Suppose the associated probabilities are $p_1, p_2, ..., p_n$. Then the expected value of x is

$$E(x) = x_1 p_1 + x_2 p_2 + ... + x_n p_n .$$

A probability distribution is given by the table below. Find the expected value.

x	3	4	5	6	7	8	9
$P(x)$	0.15	0.20	0.30	0.12	0.08	0.10	0.05

<u>Example 2:</u> Calculate the expected value of the variable of the variable representing the result of rolling a single fair die.

<u>Example 1:</u> Write the probability distribution associated with the rolling of two dice, where X represents the sum of the numbers on the dice. Draw a histogram to represent the distribution. What is the expected value of the sum when two dice are rolled?

<u>Example 2:</u> Suppose that an organization sells 1000 raffle tickets for $1 each. One ticket is for an IPod worth $200, and three tickets are for $50 gift certificates to a restaurant. Find the expected net winnings for a person who buys one ticket.

Note: Sometimes a probability experiment is called a *game*. A game with outcome variable X is called a *fair game* if $E(X)=0$.

<u>Example 3:</u> Suppose the yearly premium for a car insurance policy is $2300 for a customer in a certain category. Statisticians for the insurance company have determined that a person in this category has a 0.007 probability of having an accident that costs the insurance company $100,000 and a 0.015 probability of having an accident that costs the insurance company $30,000. What is the expected value of the insurance policy to the customer? To the insurance company?