Example 4: A pet supply company makes two types of fancy dog beds: the *Couch* and the *Throne*. Each *Couch* requires 30 minutes in the cutting department and 45 minutes in the sewing department. Each *Throne* requires 45 minutes in cutting and 75 in sewing. If the company has allocated a total of 80 hours of cutting time and 130 hours of sewing time to dog beds, how many of each type can be made?

x = number of Couches made
y = number of Thrones

	Couches	Thrones	Total Max
Cutting	30 min	45 min	80 hrs = 4800 min
Sewing	45 min	75 min	130 hrs = 7800 min

$$30x + 45y = 4800 \xrightarrow{(3)} 90x + 135y = 14400$$
$$45x + 75y = 7800 \xrightarrow{(-2)} -90x - 150y = -15600$$

$$-15y = -1200$$
$$\frac{-15y}{-15} = \frac{-1200}{-15}$$
$$y = 80$$

Put $y = 80$ into $30x + 45y = 4800$

$$30x + 45(80) = 4800$$
$$30x + 3600 = 4800$$
$$30x = 1200$$
$$x = 40$$

They can make 80 Thrones and 40 couches.

Example 6: For a certain area, when the supply of available square bales of hay was at 30,000 bales and the demand was for 26,000 bales, the price per bale was $8.50. When the supply was 25,000 bales and the demand was 23,000 bales, the price per bale rose to $10.75. Assuming that the price-supply and price-demand equations are linear, calculate:

a) the price-supply equation.
b) the price-demand equation.
c) the equilibrium price and quantity.

a) Price-supply

$(x, y) = $ (supply (in 1000s), price)

$(30, 8.5)$

$(25, 10.75)$

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10.75 - 8.5}{25 - 30}$

$m = \frac{2.25}{-5} = -0.45$

$y - y_1 = m(x - x_1)$

$y - 8.5 = -0.45(x - 30)$

$y = -0.45x + 13.5 + 8.5$

$y = -0.45x + 22$

y = price $\Rightarrow$ can use p for y:

$p = -0.45x + 22$

b) Price-demand

$(x, y) = $ (demand, price).

ordered pairs: $(26, 8.5)$
$(23, 10.75)$

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10.75 - 8.5}{23 - 26}$

$= \frac{2.25}{-3} = -0.75$

$y - y_1 = m(x - x_1)$

$y - 8.5 = -0.75(x - 26)$

$y - 8.5 = -0.75x + 19.5$

$y = -0.75x + 28 \Longrightarrow \boxed{p = -0.75x + 28}$

Price-demand equation

Equilibrium point:

Solve the system: $p = -0.75x + 28$
$p = -0.45x + 22$

(just set them equal to each other)

Example 7: After enjoying detailing cars for his friends and family for years, Frodo has decided to open his own car detailing shop. To keep his shop going, he must pay fixed costs of \$1300 per month (lease of the space, basic utilities, etc.) even if he doesn't clean any cars. He charges an average of \$200 for each detail job. Each detail job costs Frodo approximately \$20 in supplies, electricity, and water; each job also costs Frodo \$120 in terms of time. (He made \$30/hour at his old job, and he figures 4 hours to do each car.)

a) Find the cost equation and revenue equation for Frodo's business.
b) How many cars must Frodo clean each month for his business to break even?

x = number of cars detailed per month

a) Cost eqn:

Per month:

$\text{Cost} = 1300 + 140x$

Can write it as a function: $\boxed{C(x) = 1300 + 140x \quad \text{Cost equation}}$

Revenue Eqn

$\text{Revenue} = 200x$

$\boxed{R(x) = 200x \quad \text{Revenue eqn}}$

b) To find the break-even point, we must have Revenue = Cost

$$200x = 1300 + 140x$$
$$60x = 1300$$
$$x = 21.67$$

So, he needs to do at least 21.67 cars (so really 22).

Example 4: Write the augmented matrix for the system.

$$2x-3y+z=9$$
$$-x+8z=-2$$
$$4x-7y+6z=0$$

$$\left[\begin{array}{ccc|c} 2 & -3 & 1 & 9 \\ -1 & 0 & 8 & -2 \\ 4 & -7 & 6 & 0 \end{array}\right]$$

Example 5: Give the system represented by $\left[\begin{array}{ccc|c} -8 & 0 & 7 & 1 \\ 5 & 3 & -6 & 2 \\ 0 & 2 & -1 & -3 \end{array}\right]$.

$$-8x \quad + 7z = 1$$
$$5x + 3y - 6z = 2$$
$$2y - 1z = -3$$

Often we use x_1, x_2, x_3 instead of x, y, z.