7.3: Basic Counting Principles

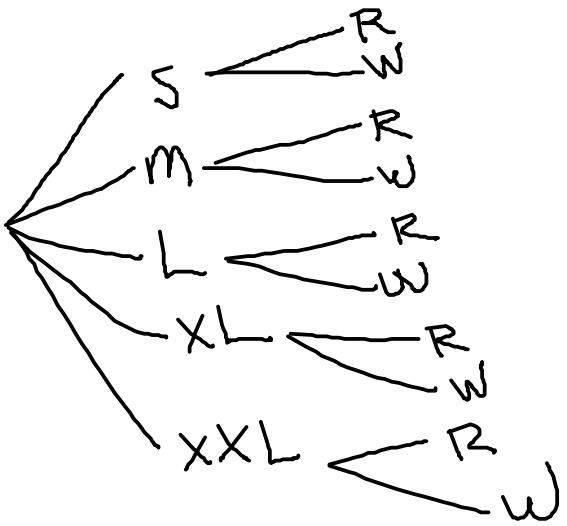
Addition Principle

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Multiplication principle for counting:

This principle is used to analyze sets which are determined by a sequence of operations.

Example 1: A company sells football jerseys. The jerseys come in sizes S, M, L, XL, and XXL. They also come in two colors: red for home games and white for away games. How many total types of jerseys does the company make?



10 types of jerseys

Multiplication Principle

Suppose that n choices must be made, with

m_1 ways to make choice 1,
m_2 ways to make choice 2,
m_3 ways to make choice 3,
.
.
.
m_n ways to make choice n.

Then there are $m_1 \cdot m_2 \cdot \ldots \cdot m_n$ ways to make the entire sequence of choices.

Example 2: How many license plate "numbers" can be formed by using a letter, followed by two digits, followed by three more letters?

$26 \cdot 10 \cdot 10 \cdot 26 \cdot 26 \cdot 26 = 45\,697\,600$

How many can be formed assuming adjacent letters and numbers must be different?

$26 \cdot 10 \cdot 9 \cdot 26 \cdot 25 \cdot 25$

How many can be formed assuming letters and numbers cannot be repeated?

$26 \cdot 10 \cdot 9 \cdot 25 \cdot 24 \cdot 23$

Products like this occur so frequently that special counting formulas and notations have been developed for them. These formulas use a function called the factorial.

The Factorial:

For a natural number (positive integer) n, $n!$ is called "n-factorial". It is defined as follows:

$n! = n(n-1)(n-2)\ldots(3)(2)(1)$

$n! = n(n-1)!$

$0! = 1$ Note, $1! = 1$

Example 3:

$6! = 6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{720}$

$\frac{8!}{7!} = \frac{8 \cdot 7 \cdot 6 \cdots 2 \cdot 1}{7 \cdot 6 \cdots 2 \cdot 1} = \frac{8 \cdot 7!}{7!} = \boxed{8}$

$\frac{97!}{3!94!} = \frac{97 \cdot 96 \cdot 95 \cdot 94!}{3 \cdot 2 \cdot 1 \cdot 94!} = \frac{97(96)(95)}{6} = \boxed{147440}$

Note: Factorials grow very rapidly!

Example 4: Compare 5!, 10!, and 15!.

$5! = 120$

$10! = 3\,628\,800$

$15! = 1.3077 \times 10^{12}$