8.2: Union, Intersecion, and Complement of Events; Odds

Unions and Intersections:

Example 1: Roll a single die.

a. What is the probability of rolling a number that is even and divisible by 3?
b. What is the probability of rolling a number that is even or divisible by 3?

E: number rolled is even
T: number rolled is divisible by 3

$E = \{2,4,6\}$ $T = \{3,6\}$

a) $E \cap T = \{6\} \Rightarrow P(E \cap T) = \boxed{\frac{1}{6}}$

b) $E \cup T = \{2,3,4,6\} \Rightarrow P(E \cup T) = \frac{4}{6} = \boxed{\frac{2}{3}}$

Probability of a Union of Two Events:

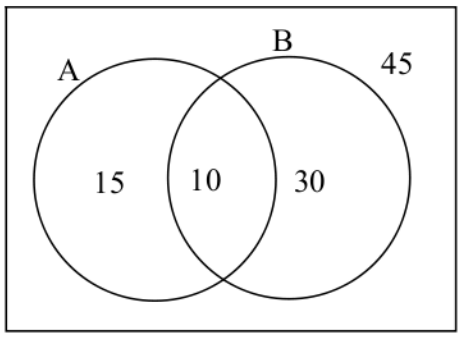
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If the two events are mutually exclusive (disjoint):

$$P(A \cup B) = P(A) + P(B)$$

$P(E \cup T)$
$= P(E) + P(T) - P(E \cap T)$
$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6}$
$= \frac{4}{6} = \boxed{\frac{2}{3}}$

Example 2: Assume that an equally likely sample space is described by the Venn diagram below.



Complements:

Probability of a complement:
$P(E') = 1 - P(E)$
$P(E) = 1 - P(E')$

Example 3: Suppose that the probability of someone voting for a certain candidate is 0.46. What is the probability of not voting for the candidate?

E: The person votes for the candidate

$P(E) = 0.46 \Rightarrow P(E') = P(E^c) = 1 - 0.46 = \boxed{0.54}$

Example 4: Roll a pair of dice. What is the probability of rolling a sum of 4 or more?

$n(S) = 36$ E: Sum is 4 or more

E^c: Sum is 2 or 3.

$E^c = \{(1,1), (2,1), (1,2)\} \Rightarrow P(E^c) = \frac{3}{36}$

$P(E) = 1 - \frac{3}{36} = \frac{33}{36} = \boxed{\frac{11}{12}}$

Odds:

Sometimes the likelihood (or unlikelihood) of an event is described using *odds* instead of probabilities.

Summary:
Probability: The event is contrasted against the whole.
Odds: The event is contrasted against the complement.

Odds for E are 3:1 (can write $\frac{3}{1}$)

Chances for E: 3

Chances for E^c: 1

Total chances: 4

$\boxed{P(E) = \frac{3}{4}}$

Converting from probability to odds:

From Probability to Odds:

- Odds for $E = \dfrac{P(E)}{P(E')}$
- Odds against $E = \dfrac{P(E')}{P(E)}$

When possible, express odds as ratios of whole numbers.

Example 5: Roll a pair of dice. What are the odds for rolling a sum of 3? What are the odds against rolling a sum of 3?

$E: \{(1,2), (2,1)\}$

Chances for E: 2

Chances against E: 34

odds for E are $\frac{2}{34} = \frac{1}{17}$ or 1:17

odds against E are $\frac{34}{2} = \frac{17}{1}$ or 17:1

Example 6: What are the odds against rolling an ace when drawing a single card from a standard deck?

4 Aces

48 non-aces

odds against getting an ace are $\frac{48}{4} = \boxed{\frac{12}{1}}$ or $\boxed{12:1}$

Example 7: Suppose that, based upon genetics, a child has a 0.08 probability of developing a certain disease. What are the odds against the child developing the disease?

Converting odds to probability:

From Odds to Probability:

If odds for an event E are $\frac{m}{n}$, (i.e. m:n) then $P(E) = \frac{m}{m+n}$.

Example 8: If the odds against a horse winning a race are 7:1, what is the probability that the horse will win?

Chances he wins: 1
Chances he loses: 7
Total chances: 8

$P(\text{win}) = \frac{1}{8}$

$P(\text{lose}) = \frac{7}{8}$

Example 9: Suppose an insurance company has used past flood data to determine that determined that the odds against a particular house flooding are 150:1. What is the probability that the house floods?