10.2: Frequency Distributions and Measures of Central Tendency

There are two basic types of numerical measures that describe data sets.

- Measures of central tendency (this section)
- Measures of dispersion (next section)

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Summation Notation: This is a compact way to write "add up the numbers $x_1, x_2, \ldots, x_n$"

(Sigma notation)

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + \ldots + x_n$$

Σ = capital Sigma

Example 1: Consider the numbers 8, 2, 6, 10, 4, 9. . Find $\sum_{i=1}^{6} x_i$ and $\sum_{i=1}^{6} x_i^2$.

$$\sum_{i=1}^{6} x_i = 8+2+6+10+4+9$$

$$\sum_{i=1}^{6} x_i^2 = 8^2+2^2+6^2+10^2+4^2+9^2$$

The Mean: (the average)

The *mean* of a set of quantitative data is equal to the sum of all the measurements in the data set divided by the total number of measurements in the set.

If $x_1, x_2, \ldots, x_n$ is a set of *n* measurements, then the *mean* is calculated by dividing the sum of the measurements by the number of measurements. The mean is sometimes known as the average.

μ = "mu"

For a population of size *n*, the mean is

$$\mu = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2 + \ldots + x_n}{n}.$$

For a sample of size *n* taken from a larger population, the mean is

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2 + \ldots + x_n}{n}.$$

Example 1: A sample has the following measurements. Find the mean.

23.1, 14.6, 21.2, 18.9, 19.3, 17.6

$$\bar{x} = \frac{23.1 + 14.6 + \ldots + 17.6}{6} = \boxed{19.1}$$

The Mean: Grouped Data:

A data set of *n* measurements is grouped into *k* classes in a frequency table. If x_i is the midpoint of the *i*th class interval and f_i is the *i*th class frequency, then the *mean* of the grouped data is approximated by

$$\mu = \frac{\sum_{i=1}^{k} x_i f_i}{n} = \frac{x_1 f_1 + x_2 f_2 + \ldots + x_k f_k}{n} \text{ for a population, or}$$

$$\bar{x} = \frac{\sum_{i=1}^{k} x_i f_i}{n} = \frac{x_1 f_1 + x_2 f_2 + \ldots + x_k f_k}{n} \text{ for a sample.}$$

In both cases, $n = \sum_{i=1}^{k} x_i f_i$ is the total number of measurements.

Example 2: Approximate the mean for the grouped data.

midpt x_i	Interval	Frequency
3	1.5–4.5	3
6	4.5–7.5	4
9	7.5–10.5	7
12	10.5–13.5	2

$n = 16$

Find midpt of 1.5–4.5

$$\frac{1.5 + 4.5}{2} = \frac{6}{2} = 3$$

$$\bar{x} = \frac{3(3) + 4(6) + 7(9) + 2(12)}{16} = \boxed{7.5}$$

Note: The same data can be represented as

Interval	Frequency
2–4	3
5–7	4
8–10	7
~~12–13~~ 11–13	2

The median:

Sometimes the mean can be misleading for a data set. Suppose that a math class had 7 students with test scores (out of a possible 100) of 88, 99, 7, 78, 89, 94, and 76.

$\mu = 75.86$

Note: all but 1 student scored above 75. (all but 1 are above average)

The 7 pulls the mean down.

The *median* is unaffected by extreme values. Essentially it is the "middle" of the data set.

To find the median, you'll need to sort the data in numerical order.

> The Median (Ungrouped Data):
>
> - If the number of measurements is odd, the median is the middle measurement when the measurements are arranged in descending or ascending order.
> - If the number of measurements is even, the median is the mean of the two middle measurements when the measurements are arranged in descending or ascending order.

Example 3: Find the median of the test scores 88, 99, 7, 78, 89, 94, and ~~75.~~ 76

7 76 78 88 89 94 99

median

$\boxed{\text{The median is 88.}}$

The mode:

> The Mode:
>
> The *mode* is the most frequently occurring measurement in a data set. There may be a unique mode, several modes, or no mode.

Example 4: Find the median and mode for the following data sets.

a. {~~4~~, ~~5~~, ~~5~~, ~~5~~, 5, ~~6~~, ~~7~~, ~~8~~, ~~12~~}

$\boxed{\text{median} = 5}$ $\boxed{\text{Mode} = 5}$

b. {~~1~~, ~~2~~, ~~3~~, ~~3~~, 3, 5, ~~7~~, ~~7~~, ~~7~~, ~~23~~}

$\text{median} = \frac{3+5}{2} = \frac{8}{2} = \boxed{4}$ Modes: $\boxed{3, 7}$ (bimodal data set)

c. {1, 3, 5, 6, 7, 9, 11, 15}

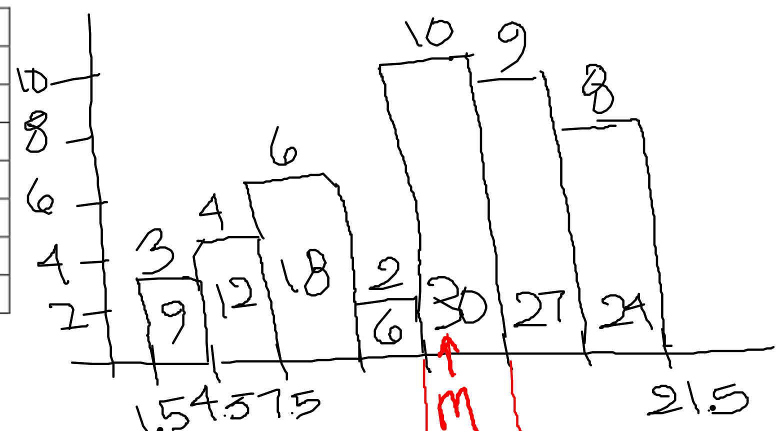
$\text{median} = \boxed{6.5}$ $\boxed{\text{No mode}}$

> The Median (Grouped Data):
>
> For grouped data, the median is the value of the variable that divides the area of the histogram into two equal portions.
> (So the area to the left of the median is equal to the area to the right of the median.)

Example 5: Approximate the median of the grouped data.

Interval	Frequency
1.5–4.5	3
4.5–7.5	4
7.5–10.5	6
10.5–13.5	2
13.5–16.5	10
16.5–19.5	9
19.5–21.5	8

$\Sigma: n = 42$



3 3 3

13.5 16.5 $16.5 - m$

$m - 13.5$

Total Area:

Each bar is 3 units wide

$A = 3(3) + 3(4) + 3(6) + 3(2) + 3(10) + 3(9) + 3(8) = 3(42) = 126$

$= 3(3+4+6+2+10+9+8)$

Half of 126 is 63.

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