Homework Qs

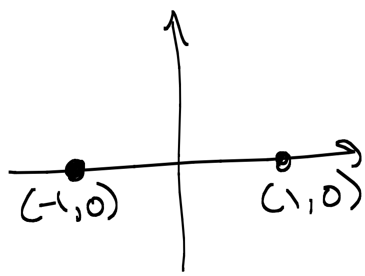
Note Title 1/29/2019

1.3 #99 Determine $\cos[(2n+1)\cdot 90^\circ]$, where n is any integer.

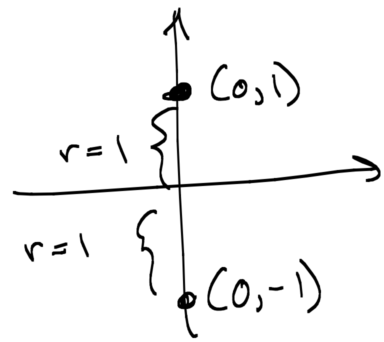
<u>Note</u>: if n is an integer, then $2n$ is <u>even</u>.
Also $2n+1$ is <u>odd</u>.
So $(2n+1)\cdot 90^\circ$ is equivalent to saying "odd multiples of 90°".

in either of these locations, $\cos\theta$ is $\frac{x}{r}$, so either $\frac{0}{1} = 0$ or $\frac{0}{1} = 0$

Even multiples of 90° put us where?



odd multiples of 90° put us where?



1.2 #25 2 angles of a triangle are $147^\circ 12'$ and $30^\circ 19'$. Find 3rd angle.

$$\begin{array}{r} 147^\circ 12' \\ +\ 30^\circ 19' \\ \hline 177^\circ 31' \end{array}$$

$$\begin{array}{r} 180^\circ 00' \\ -\ 177^\circ 31' \\ \hline \end{array} \Rightarrow \begin{array}{r} 179^\circ 60' \\ -\ 177^\circ 31' \\ \hline 2^\circ 29' \end{array}$$

<u>Ex</u>: suppose 2 angles are $53^\circ 34'$, $61^\circ 42'$

$$\begin{array}{r} 53^\circ 34' \\ 61^\circ 42' \\ \hline 114^\circ 76' \end{array} \Rightarrow 115^\circ 16'$$

$$\begin{array}{r} 180^\circ 0' \\ -\ 115^\circ 16' \end{array}$$ (similar)

Section 1.4 **Using the Definitions of the Trigonometric Functions**

We saw in the previous section that some of the trigonometric functions are reciprocals of each other.

Multiplying reciprocals together always results in a value of 1.

$(sin\,\theta)(csc\,\theta) = \frac{y}{r} \cdot \frac{r}{y} = \frac{yr}{\cancel{yr}} = 1.$

An identity is a mathematical statement that is true for all values of the variable.

This leads us to what we call the **reciprocal identities.**

$\sin\theta = \frac{1}{\csc\theta}$	$\cos\theta = \frac{1}{\sec\theta}$ $\frac{x}{r} = \frac{1}{r/x} \Rightarrow \frac{x}{r} = 1 \cdot \frac{x}{r}$	$\tan\theta = \frac{1}{\cot\theta}$
$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$

Example 1: Find each function value.

a) Find $\tan\theta$, given that $\cot\theta = 4$.

$\cot\theta = 4 = \frac{4}{1}$

$\tan(\theta) = \boxed{\frac{1}{4}}$

b) Find $\sec\theta$, given that $\cos\theta = -\frac{2}{\sqrt{20}}$

$\sec\theta = -\frac{\sqrt{20}}{2} = -\frac{\sqrt{4 \cdot 5}}{2}$

$= -\frac{\sqrt{4}\sqrt{5}}{2} = -\frac{\cancel{2}\sqrt{5}}{\cancel{2}}$

$= \boxed{-\sqrt{5}}$

c) Find $\cos\theta$, given that $\sec\theta = 9.80425133$

$\cos\theta = \frac{1}{9.80425133} \approx \boxed{0.1019965693}$

Reciprocal Key: $\boxed{x^{-1}}$ or $\boxed{1/x}$

NOTE: Reciprocals always have the same sign.

Determining the signs of the trigonometric functions of nonquadrantal angles r always positive!

QII QI

$\tan\theta = \frac{y}{x} = \frac{(+)}{(-)} \Rightarrow (-)$

$\sin\theta, \csc\theta$	positive
$\cos\theta, \sec\theta$	negative
$\tan\theta, \cot\theta$	negative

$(-, +)$

$\sin\theta = \frac{y}{r} = \frac{(+)}{(+)} \Rightarrow (+)$

$\cos\theta = \frac{x}{r} = \frac{(-)}{(+)} \Rightarrow (-)$

$\sin\theta, \csc\theta$	positive
$\cos\theta, \sec\theta$	+ positive
$\tan\theta, \cot\theta$	positive

$(+, +)$ (x, y) $(+)$ r

$\cos\theta = \frac{x}{r} = \frac{+}{+} = (+)$

$\sin\theta = \frac{y}{r} = \frac{+}{+} = (+)$

$(+, -)$

$\sin\theta, \csc\theta$	negative
$\cos\theta, \sec\theta$	negative
$\tan\theta, \cot\theta$	positive

$(-, -)$

$\tan\theta = \frac{y}{x} = \frac{(-)}{(-)} \Rightarrow (+)$ Q III

QIV

$\sin\theta, \csc\theta$	negative
$\cos\theta, \sec\theta$	positive
$\tan\theta, \cot\theta$	negative

$\tan\theta = \frac{y}{x} = \frac{(-)}{(+)} \Rightarrow (-)$

Example 2: Identify the quadrant(s) of an angle θ that satisfies the given conditions.

a) $\cos\theta < 0, \quad \sin\theta < 0$

Quadrant III

b) $\cos\theta > 0, \quad \sec\theta > 0$

Quadrant I or IV

c) $\cot\theta < 0, \quad \sec\theta < 0$

$\tan\theta < 0$, $\cos\theta < 0$

$\Downarrow$ x is neg

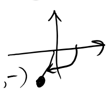
So y is pos since x is neg.

$(-, +)$

Quadrant II

Example 3: Find the signs of the six trigonometric functions for the given angle.

a) $-115°$



$(-, -)$

sin $(-115°)$ $-$

cos $(-115°)$ $-$

tan $(-115°)$ $+$ $\frac{y}{x} \Rightarrow \frac{(-)}{(-)}$

cot $(-115°)$ $+$

sec $(-115°)$ $-$

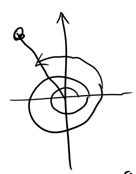
csc $(-115°)$ $-$

b) $855°$

sin $855°$ $+$

cos $855°$ $-$

tan $855°$ $-$ $\frac{y}{x} = \frac{(+)}{(-)}$



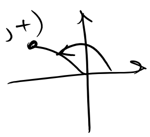
cot $855°$ $-$

sec $855°$ $-$

csc $855°$ $+$

$855° - 360° - 360° = 135°$

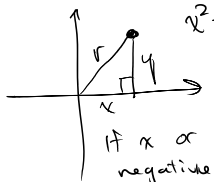
$(-, +)$



Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\quad$ $\tan^2\theta + 1 = \sec^2\theta$ $\quad$ $1 + \cot^2\theta = \csc^2\theta$

We know form the Pythagorean Theorem that $y^2 + x^2 = r^2$.



$x^2 + y^2 = r^2$

If x or y is negative, we still get $x^2 + y^2 = r^2$

$y^2 + x^2 = r^2$

Divide both sides by r^2: $\frac{y^2}{r^2} + \frac{x^2}{r^2} = \frac{r^2}{r^2}$

$\left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2 = 1$

$\boxed{(\sin\theta)^2 + (\cos\theta)^2 = 1}$

It is standard notation to write $\sin^2\theta$ instead of $(\sin\theta)^2$.

Main/primary Pythagorean Identity: $\sin^2\theta + \cos^2\theta = 1$

Divide both sides by $\cos^2\theta$: $\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

Note: $\frac{\sin\theta}{\cos\theta} = \frac{y/r}{x/r} = \frac{y}{\cancel{r}} \cdot \frac{\cancel{r}}{x} = \frac{y}{x} = \tan\theta \Rightarrow$ $\boxed{\tan^2\theta + 1 = \sec^2\theta}$

Start again with $\sin^2\theta + \cos^2\theta = 1$

Divide by $\sin^2\theta$: $\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$\boxed{1 + \cot^2\theta = \csc^2\theta}$

NOTE: It is important to be able to transform these identities into their equivalent forms.

Quotient Identities

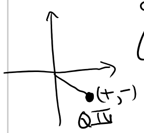
$\frac{\sin\theta}{\cos\theta} = \tan\theta$ $\quad$ $\frac{\cos\theta}{\sin\theta} = \cot\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{y/r}{x/r} = \frac{y}{\cancel{r}} \cdot \frac{\cancel{r}}{x} = \frac{y}{x} = \tan\theta$

Using Identities to find Function Values

Example 4: Find $\sin\theta$, given that $\cos\theta = \frac{4}{5}$ and θ is in quadrant IV.

Using Pythagorean Identity



$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{4}{5}\right)^2 + \sin^2\theta = 1$

$\frac{16}{25} + \sin^2\theta = 1$

$\sin^2\theta = 1 - \frac{16}{25}$

$(\sin\theta)^2 = \frac{9}{25}$

$\sin\theta = \pm\sqrt{\frac{9}{25}}$

$\sin\theta = \pm\frac{3}{5}$

Quadrant IV, so choose negative value. $\boxed{\sin\theta = -\frac{3}{5}}$

Note:

$x^2 = 9$

$x = \pm\sqrt{9}$

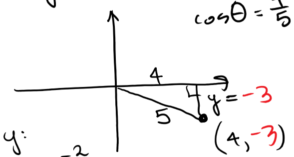
$x = \pm 3$

$x^2 = 7$

$x = \pm\sqrt{7}$

Using a triangle

$\cos\theta = \frac{4}{5} = \frac{x}{r}$



Find y:

$4^2 + y^2 = 5^2$

$16 + y^2 = 25$

$y^2 = 25 - 16 = 9$

$y = \pm\sqrt{9} = \pm 3$

QIV, so choose negative ($y<0$ in QIV)

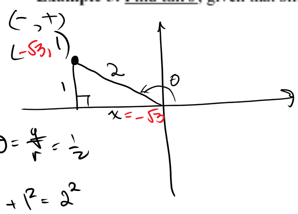
$y = -3$

so $\sin\theta = \frac{y}{r}$

$= \frac{-3}{5}$

$\boxed{\sin\theta = -\frac{3}{5}}$

Example 5: Find $\tan\theta$, given that $\sin\theta = \frac{1}{2}$ and θ is in quadrant II.



$\sin\theta = \frac{y}{r} = \frac{1}{2}$

$x^2 + 1^2 = 2^2$

$x^2 + 1 = 4$

$x^2 = 3$

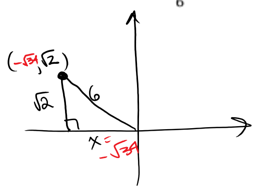
$x = \pm\sqrt{3}$

QII, so choose $x = -\sqrt{3}$

Then, $\tan\theta = \frac{y}{x} = \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{-\frac{\sqrt{3}}{3}}$

Example 6: Find the five remaining trigonometric function values for each angle θ.

a) $\sin\theta = \frac{\sqrt{2}}{6}$, and $\cos\theta < 0$



$\sin\theta$ is positive, and $\cos\theta$ is negative, so Quadrant II.

$x^2 + (\sqrt{2})^2 = 6^2$

$x^2 + 2 = 36$

$x^2 = 34$

$x = \pm\sqrt{34}$

Q2, so choose $x = -\sqrt{34}$

$\cos\theta =$ ____

$\tan\theta = \boxed{-\frac{\sqrt{17}}{17}}$

$\tan\theta = \frac{y}{x} = \frac{\sqrt{2}}{-\sqrt{34}} = -\sqrt{\frac{2}{34}}$

$= -\sqrt{\frac{1}{17}} = -\frac{1}{\sqrt{17}} = -\frac{\sqrt{17}}{17}$

$\cot\theta =$ ____

OR

$\tan\theta = -\frac{\sqrt{2}}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}}$

$= -\frac{\sqrt{68}}{34}$

$= -\frac{\sqrt{4\cdot 17}}{34}$

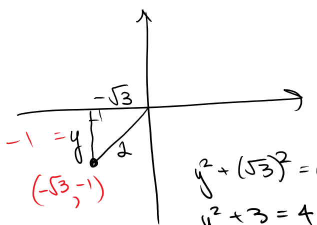
$= -\frac{\cancel{2}\sqrt{17}}{\cancel{34}\,17}$

$= -\frac{\sqrt{17}}{17}$

$\sec\theta =$ ____

$\csc\theta =$ ____

b) $\cos\theta = -\frac{\sqrt{3}}{2}$, and θ is in quadrant III



$y^2 + (\sqrt{3})^2 = (2)^2$

$y^2 + 3 = 4$

$y^2 = 1$

$y = \pm\sqrt{1} = \pm 1$

Q III, so choose $y = -1$

$\sin\theta = \boxed{-\frac{1}{2}}$

$\sin\theta = \frac{y}{r} = \frac{-1}{2}$

$\tan\theta = \boxed{\frac{\sqrt{3}}{3}}$

$\tan\theta = \frac{y}{x} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\cot\theta = \boxed{\sqrt{3}}$

$\cot\theta = \frac{x}{y} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$

$\sec\theta = \boxed{-\frac{2\sqrt{3}}{3}}$

$\sec\theta = \frac{r}{x} = \frac{2}{-\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

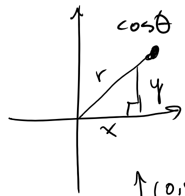
$\csc\theta = \boxed{-2}$

$\csc\theta = \frac{r}{y} = \frac{2}{-1} = -2$

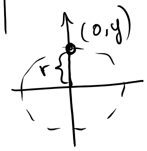
The Range Values of the six trigonometric functions (Output values of the functions)

$\sin\theta = \frac{y}{r}$

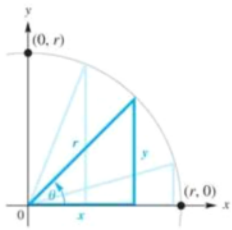
$\cos\theta = \frac{x}{r}$

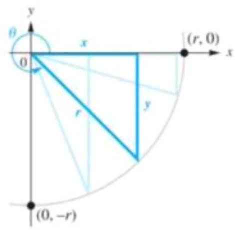


Trigonometric Function of θ	Range
$\sin\theta, \cos\theta$ $\frac{y}{r}, \frac{x}{r}$	$[-1, 1]$
$\csc\theta, \sec\theta$ $\frac{r}{y}, \frac{r}{x}$	$(-\infty, -1] \cup [1, \infty)$
$\tan\theta, \cot\theta$ $\frac{y}{x}$	$(-\infty, \infty)$



Since r always represents the longest side of the triangle,





The sides x and y can vary greatly in relationship to each other.

$y = x$ OR $y < x$ OR $y > x$

Example 7: Decide whether each statement is possible or impossible for some angle θ.

a) $\sin\theta = 3$

$\boxed{\text{impossible}}$

$-1 \le \sin\theta \le 1$

b) $\cos\theta = -0.96$

$\boxed{\text{possible}}$

c) $\csc\theta = 100$

$\sin\theta = \frac{1}{100}$

$\boxed{\text{possible}}$

d) $\cos\theta = -2$ *and* $\sec\theta = -\frac{1}{2}$

$\boxed{\text{impossible}}$

$-1 \le \cos\theta \le 1$

e) $\csc\theta = -2$ *and* $\sin\theta = -\frac{1}{2}$

$\boxed{\text{possible}}$

HWQ: What is wrong with:
#25 Find $\sec\theta$, given that $\cos\theta = \frac{3}{2}$.

$\cos\theta = 1\frac{1}{2}$

this is impossible

#83 Given $\sin\theta = 1$, find the other trig functions.

$\sin\theta = \frac{y}{r} = 1 = \frac{1}{1}$