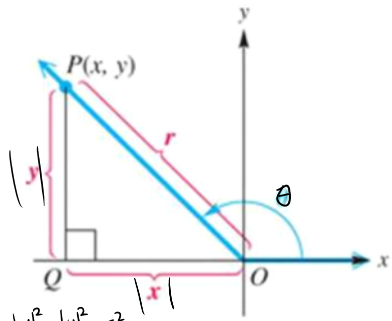
θ: Greek letter Theta

1.3.1



θ is any angle in standard position

P is any point (x, y) on the terminal side of the angle

r is the distance from the origin to the point P

Using the distance formula, we know that

$$r = \sqrt{x^2 + y^2} \quad and \quad r > 0$$

Pythagorean theorem: $a^2 + b^2 = c^2$

$|x|^2 + |y|^2 = r^2$

$x^2 + y^2 = r^2 \Rightarrow r = \pm\sqrt{x^2+y^2}$. r is a distance, so we choose the positive. $r = \sqrt{x^2+y^2}$

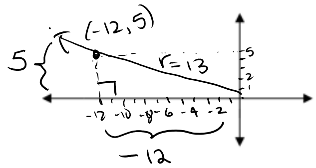
The six trigonometric functions are defined as follows

(sine)	(cosine)	(tangent)
$\sin\theta = \frac{y}{r}$	$\cos\theta = \frac{x}{r}$	$\tan\theta = \frac{y}{x}$ $(x \neq 0)$
$\csc\theta = \frac{r}{y}$ $(y \neq 0)$	$\sec\theta = \frac{r}{x}$ $(x \neq 0)$	$\cot\theta = \frac{x}{y}$ $(y \neq 0)$
(cosecant)	(secant)	(cotangent)

"should" be $\sin(\theta) = \frac{y}{r}$

Example 1:

The terminal side of an angle θ in standard position passes through the point (−12, 5). Find the values of the six trigonometric functions.



$r^2 = 5^2 + 12^2$

$r^2 = 25 + 144$

$r^2 = 169$

$r = \pm\sqrt{169} = \pm 13$. Choose $r = 13$

$\sin\theta = \frac{y}{r} = \boxed{\frac{5}{13}}$

$\cos\theta = \frac{x}{r} = \frac{-12}{13} = \boxed{-\frac{12}{13}}$

$\csc\theta = \frac{r}{y} = \boxed{\frac{13}{5}}$

$\sec\theta = \frac{r}{x} = \boxed{-\frac{13}{12}}$

$\tan\theta = \frac{y}{x} = \frac{5}{-12} = \boxed{-\frac{5}{12}}$

$\cot\theta = \frac{x}{y} = \frac{-12}{5} = \boxed{-\frac{12}{5}}$

$\boxed{1.3.2}$

Example 2:

The terminal side of an angle θ in standard position passes through the point $\left(-2\sqrt{3},-2\right)$. Find the values of the six trigonometric functions.

$r^2 = x^2 + y^2$

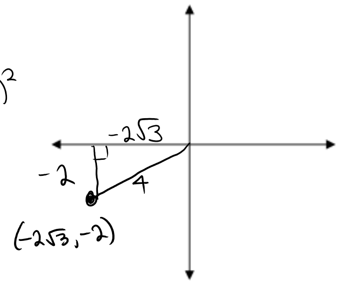
$= (-2\sqrt{3})^2 + (-2)^2$

$= 4 \cdot 3 + 4$

$= 12 + 4$

$= 16$

$r = \sqrt{16} = 4$



$\cot(\theta) = \frac{x}{y} = \frac{-\cancel{2}\sqrt{3}}{-\cancel{2}} = \boxed{\sqrt{3}}$

$\sin(\theta) = \frac{y}{r} = \frac{-2}{4} = \boxed{-\frac{1}{2}}$

$\cos(\theta) = \frac{x}{r} = \frac{-\cancel{2}^{1}\sqrt{3}}{\cancel{4}_{2}} = \boxed{-\frac{\sqrt{3}}{2}}$

$\tan(\theta) = \frac{y}{x} = \frac{-\cancel{2}^{1}}{-\cancel{2}\sqrt{3}} = \frac{1}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

$\csc(\theta) = \frac{r}{y} = \frac{4}{-2} = \boxed{-2}$

$\sec(\theta) = \frac{r}{x} = \frac{\cancel{4}^{2}}{-\cancel{2}_{1}\sqrt{3}} = -\frac{2}{\sqrt{3}}$

$= -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \boxed{-\frac{2\sqrt{3}}{3}}$

$\sin\theta =$ $\cos\theta =$

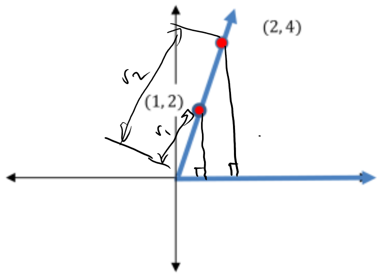
$\csc\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

1.3.3

NOTE: We can pick **ANY** point on the terminal side to find the values of the six functions.

For the following angle θ , show that the points $(1, 2)$ *and* $(2, 4)$ will give the same values for the six functions.



The terminal side of an angle θ in standard position passes through the point $(1, 2)$. Find the function values.

$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

The terminal side of an angle θ in standard position passes through the point $(2, 4)$. Find the function values.

$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

1, 3, 4

Example 3:

An equation of the terminal side of an angle θ in standard position is given with a restriction on x. Sketch the least positive such angle θ, and find the value of the six trigonometric functions of θ.

$-5x - 3y = 0, \qquad x \leq 0$

Write in slope-intercept form

$y = mx + b.$

Solve for y:

$-3y = 5x$

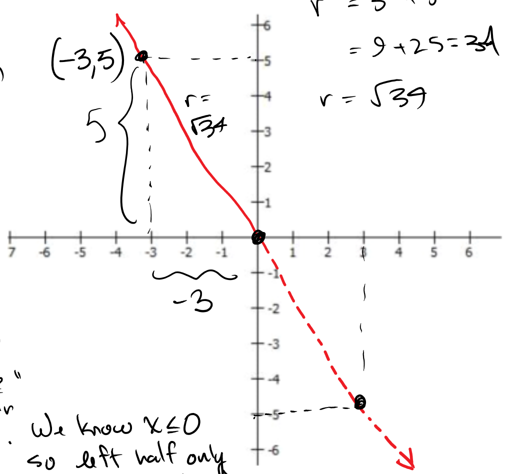
$\frac{-3y}{-3} = \frac{5x}{-3}$

$y = -\frac{5}{3}x$

$y = -\frac{5}{3}x + 0$

Slope: $m = -\frac{5}{3}$ "$\frac{\text{rise}}{\text{run}}$"

y-intercept: $b = 0$



$r^2 = 3^2 + 5^2$

$= 9 + 25 = 34$

$r = \sqrt{34}$

We know $x \leq 0$
so left half only
(Quadrant II)

$\sin\theta =$ $\qquad$ $\cos\theta =$

$\csc\theta =$ $\qquad$ $\sec\theta =$

$\tan\theta =$ $\qquad$ $\cot\theta =$

1.3.5

The trigonometric function values of the quadrantal angles.

$\cos(90^\circ) = \frac{x}{r} = \frac{0}{1} = \boxed{0}$

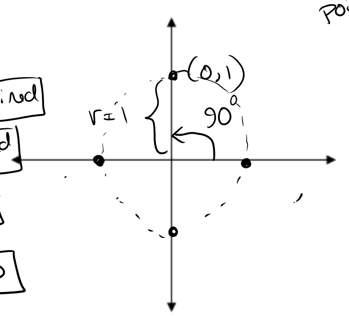
$\sin(90^\circ) = \frac{y}{r} = \frac{1}{1} = \boxed{1}$

$\tan(90^\circ) = \frac{y}{x} = \cancel{\frac{1}{0}}\ \boxed{\text{undefined}}$

$\sec(90^\circ) = \frac{r}{x} = \cancel{\frac{1}{0}}\ \boxed{\text{undefined}}$

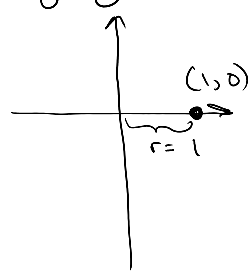
$\csc(90^\circ) = \frac{r}{y} = \frac{1}{1} = \boxed{1}$

$\cot(90^\circ) = \frac{x}{y} = \frac{0}{1} = \boxed{0}$



r = distance from the point to the origin

$\theta = 0^\circ$



θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
0°						
90°						
180°						
270°						

NOTE: Coterminal angles have the same trigonometric function values.