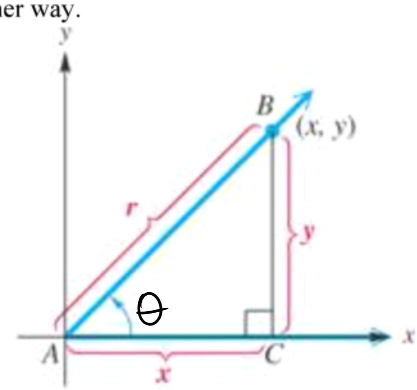
2.1.1

Section 2.1 Trigonometric Functions of Acute Angles

In section 1.3, we used a point on the terminal side of the angle to define the trigonometric functions.

In this section, we will approach them another way.



Acute angle:

θ is an acute angle if

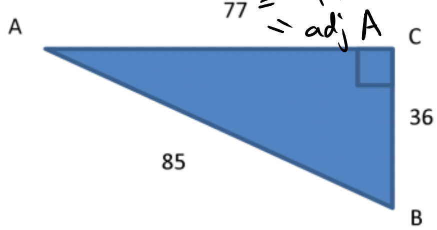
$0^\circ < \theta < 90^\circ$

Right Triangle-Based Definitions. For an acute angle A,

$$\sin A = \frac{y}{r} = \frac{\text{side opposite } A}{\text{hypotenuse}} \qquad \cos A = \frac{x}{r} = \frac{\text{side adjacent to } A}{\text{hypotenuse}} \qquad \tan A = \frac{y}{x} = \frac{\text{side opposite } A}{\text{side adjacent to } A}$$

$$\csc A = \frac{r}{y} = \frac{\text{hypotenuse}}{\text{side opposite } A} \qquad \sec A = \frac{r}{x} = \frac{\text{hypotenuse}}{\text{side adjacent to } A} \qquad \cot A = \frac{x}{y} = \frac{\text{side adjacent to } A}{\text{side opposite } A}$$

Example 1:



Mnemonic device:
"SOH-CAH-TOA"

$\sin\theta \Rightarrow \frac{\text{Opp}}{\text{Hyp}}$

$\cos\theta \Rightarrow \frac{\text{Adj}}{\text{Hyp}}$

$\tan\theta \Rightarrow \frac{\text{Opp}}{\text{Adj}}$

$\sin A = \frac{\text{opp}}{\text{hyp}} = \boxed{\frac{36}{85}}$

$\cos A = \frac{\text{adj}}{\text{hyp}} = \boxed{\frac{77}{85}}$

$\tan A = \frac{\text{opp}}{\text{adj}} = \boxed{\frac{36}{77}}$

$\cot A = \frac{\text{adj}}{\text{opp}} = \frac{77}{36}$

$\sin B = \frac{\text{opp}}{\text{hyp}} = \boxed{\frac{77}{85}}$

$\cos B = \frac{\text{adj}}{\text{hyp}} = \boxed{\frac{36}{85}}$

$\tan B = \frac{\text{opp}}{\text{adj}} = \boxed{\frac{77}{36}}$

2 angles are complements of each other if their sum is 90°.

Note: $A + B = 90^\circ$

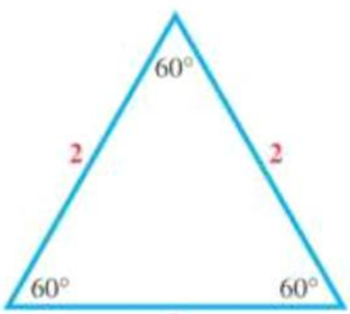
Example 2: Suppose ABC is a right triangle with sides of lengths a, b, c, and right angle at C. Find the unknown side length using the Pythagorean theorem, and then find the values of the six trigonometric functions for angle B.

$a = 6, \ \ c = 7$ b = ________ $\sin B =$ $\csc B =$

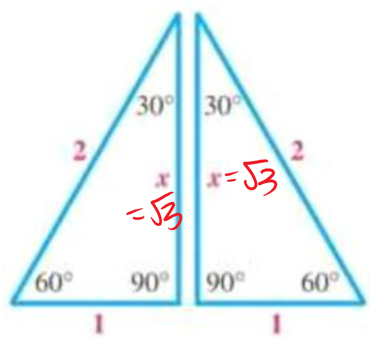
$\cos B =$ $\sec B =$

$\tan B =$ $\cot B =$

Trigonometric Function Values of Special Angles



Equilateral triangle



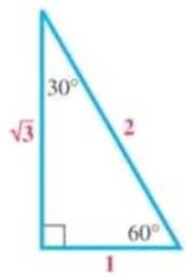
30°– 60° right triangle

$1^2 + x^2 = 2^2$

$1 + x^2 = 4$

$x^2 = 3$

$x = \pm\sqrt{3}$. Choose + because it's a triangle



sin 30° = $\boxed{\frac{1}{2}}$ **csc 30° =**

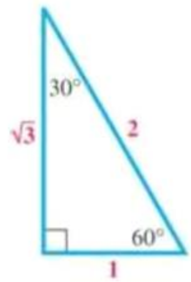
$\frac{opp}{hyp}$

cos 30° = $\boxed{\frac{\sqrt{3}}{2}}$ **sec 30° =**

$\frac{adj}{hyp}$

tan 30° = $\boxed{\frac{1}{\sqrt{3}}}$ **cot 30° =**

$\frac{opp}{adj}$



sin 60° = **csc 60° =**

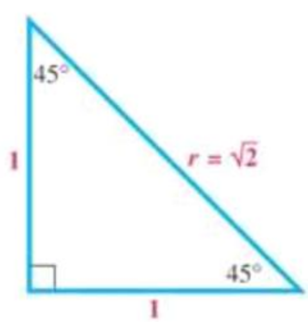
$\frac{opp}{hyp} = \boxed{\frac{\sqrt{3}}{2}}$

cos 60° = $\boxed{\frac{1}{2}}$ **sec 60° =**

$\frac{adj}{hyp}$

tan 60° = $\boxed{\sqrt{3}}$ **cot 60° =**

$\frac{opp}{adj} = \frac{\sqrt{3}}{1}$



45°– 45° right triangle

45°-45°-90°

sin 45° = $\frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$ **csc 45° =**

$\frac{opp}{hyp}$

cos 45° = $= \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$ **sec 45° =**

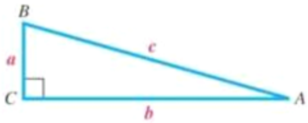
$\frac{adj}{hyp}$

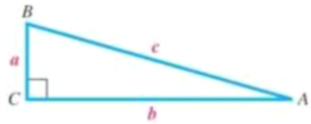
tan 45° = $\boxed{1}$ **cot 45° =**

$\frac{opp}{adj} = \frac{1}{1}$

Cofunction Identities

A + B =





$\sin A =$

$\sin B =$

$\cos A =$

$\cos B =$

Ex: $\sin(70^\circ) = \cos(20^\circ)$

Cofunction Identities

For any acute angle *A*, cofunction values of complementary angles are equal.

$$\sin A = \cos(90^\circ - A) \quad \sec A = \csc(90^\circ - A) \quad \tan A = \cot(90^\circ - A)$$
$$\cos A = \sin(90^\circ - A) \quad \csc A = \sec(90^\circ - A) \quad \cot A = \tan(90^\circ - A)$$

Example 3: Write each function in terms of its cofunction.

a) $\sin 9^\circ$

$\sin(9^\circ) = \cos(81^\circ)$

b) $\cot 76^\circ$

$\cot(76^\circ) = \tan(14^\circ)$

c) $\csc 60^\circ$

$\csc 60^\circ = \sec 30^\circ$