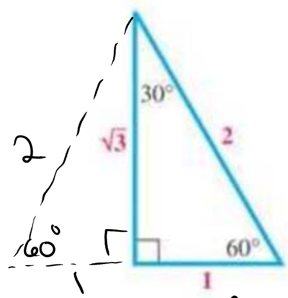
Section 2.2 Trigonometric Functions of Non-Acute Angles

Make sure that you know the special triangles!



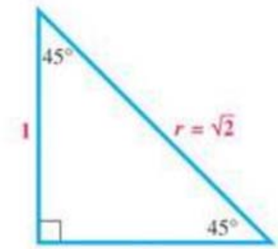
$y^2 + 1^2 = 2^2$

$y^2 = 3$

$y = \pm\sqrt{3}$

$y = \sqrt{3}$

30°-60°-90° right triangle



45°–45° right triangle

45°-45°-90° right triangle

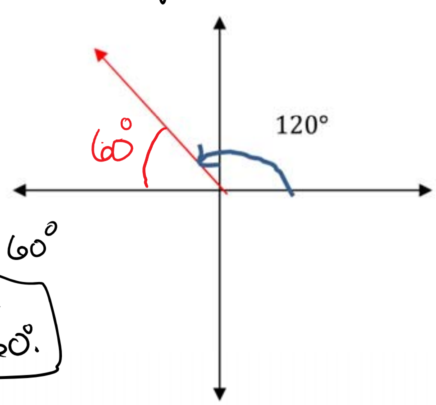
Reference Angles

Recall

A **reference angle** is the positive acute angle made by the terminal side and the **x-axis.**

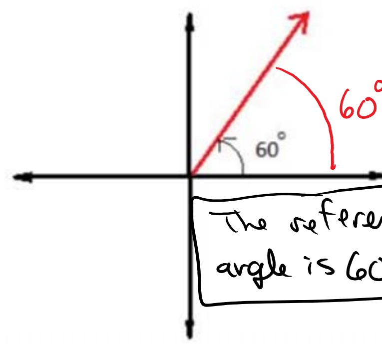
An angle θ is acute if $0° < \theta < 90°$

Find the reference angle for each angle:



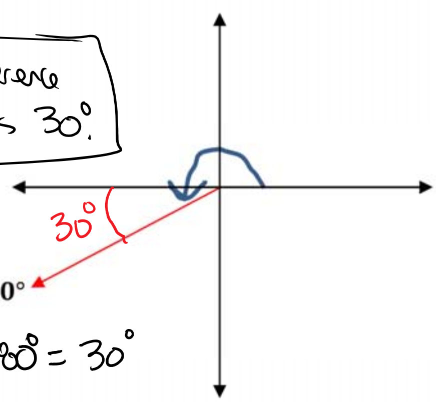
$180° - 120° = 60°$

The reference angle is 60°.



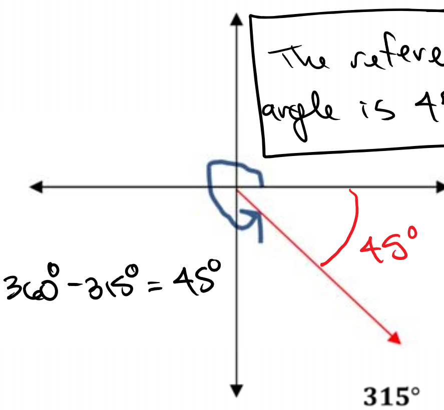
The reference angle is 60°

The reference angle is 30°.



$210° - 180° = 30°$

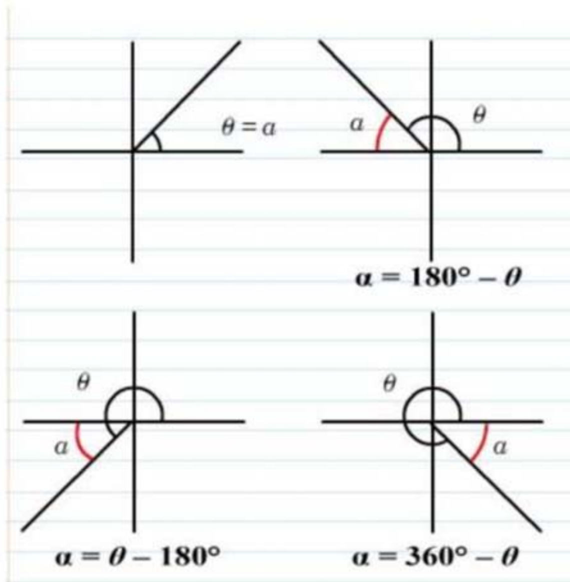
The reference angle is 45°



$360° - 315° = 45°$

2.2.2

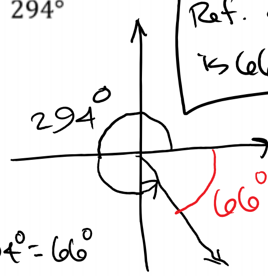
For any angle θ (between $0°$ and $360°$), we can find the reference angle α by using the following table.



Example 1: Find the reference angle for the following angles.

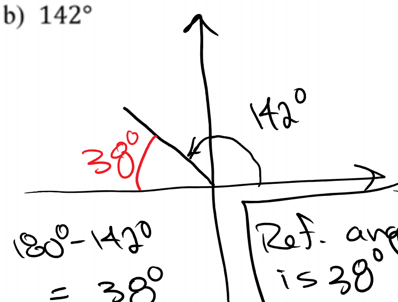
a) 294°

Ref. angle is 66°



$360° - 294° = 66°$

b) 142°

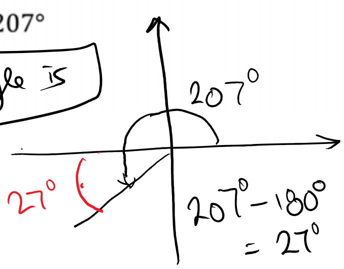


$180° - 142°$
$= 38°$

Ref. angle is 38°

c) 207°

Ref. angle is 27°



$207° - 180°$
$= 27°$

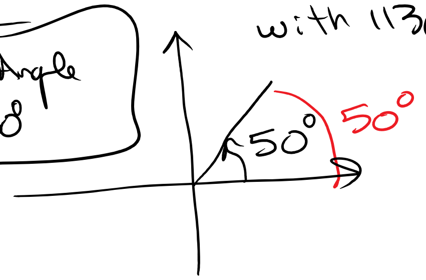
d) 1130°

$\frac{1130°}{360°} \approx 3.13$

$1130° - 3(360°)$
$= 50°$

So 50° is coterminal with 1130°

Ref. Angle is 50°

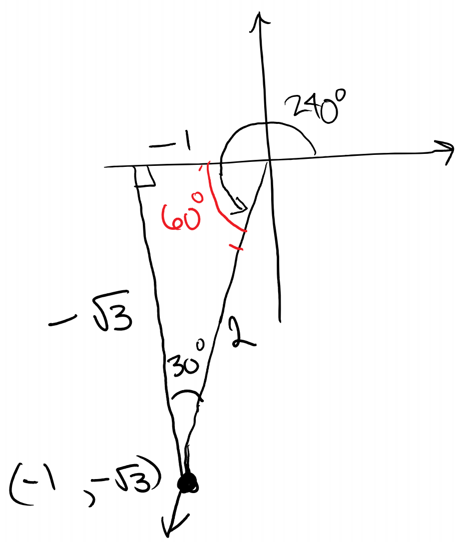


2.2.3

Finding Trigonometric Function Values for Any Nonquadrantal Angle θ
Step 1 If θ > 360°, or if θ < 0°, then find a coterminal angle by adding or subtracting 360° as many times as needed to get an angle greater than 0° but less than 360°.
Step 2 Find the reference angle θ'.
Step 3 Find the trigonometric function values for reference angle θ'.
Step 4 Determine the correct signs for the values found in Step 3. (Use the table of signs in **Section 1.4,** if necessary.) This gives the values of the trigonometric functions for angle θ.

We can use the reference angle to find values of the six trigonometric functions for non-acute angles.

Example 2: Find the values of the six trigonometric functions for 240°.



sin 240° = $\boxed{-\frac{\sqrt{3}}{2}}$

$\frac{y}{r} = \frac{-\sqrt{3}}{2}$

csc 240° = $\boxed{-\frac{2\sqrt{3}}{3}}$

$\frac{r}{y} = \frac{2}{-\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

cos 240° = $\boxed{-\frac{1}{2}}$

$\frac{x}{r} = \frac{-1}{2}$

sec 240° = $\boxed{-2}$

(reciprocal of cosine)

tan 240° = $\boxed{\sqrt{3}}$

$\frac{y}{x} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$

cot 240° = $\boxed{\frac{\sqrt{3}}{3}}$

$\frac{x}{y} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

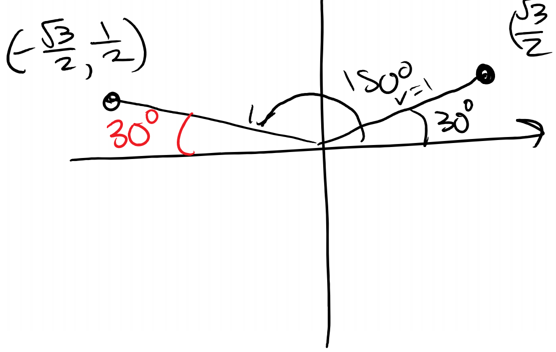
Example 3: Find the values of the six trigonometric functions for 150°.

Here, r = 1

cos 30° sin 30°

↓ ↓

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$



sin 150° = $\boxed{\frac{1}{2}}$

csc 150° = $\boxed{2}$

cos 150° = $\boxed{-\frac{\sqrt{3}}{2}}$

sec 150° = $\boxed{-\frac{2\sqrt{3}}{3}}$

$-\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

tan 150° = $\boxed{-\frac{\sqrt{3}}{3}}$

$\tan 150° = \frac{\sin 150°}{\cos 150°} = \frac{1/2}{-\sqrt{3}/2}$

$= \frac{1}{\cancel{2}} \cdot \left(-\frac{\cancel{2}}{\sqrt{3}}\right) = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

cot 150° = $\boxed{-\sqrt{3}}$

$\frac{\cos 150°}{\sin 150°} = -\frac{3}{\sqrt{3}}$

$= -\frac{\cancel{3}\sqrt{3}}{\cancel{3}}$

$= -\sqrt{3}$

2.2.4

e) -1637°

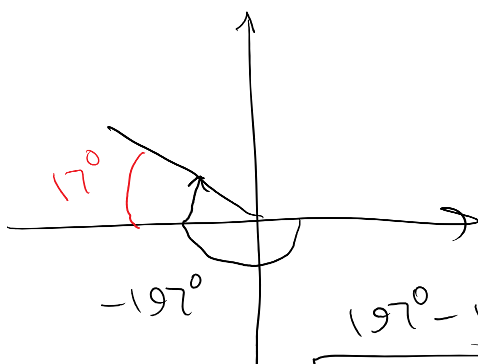
$\frac{1637^\circ}{360^\circ} \approx 4.5472$

$1637^\circ - 4(360^\circ) = 197^\circ$

It's negative, so

-1637° and -197°

are coterminal angles.

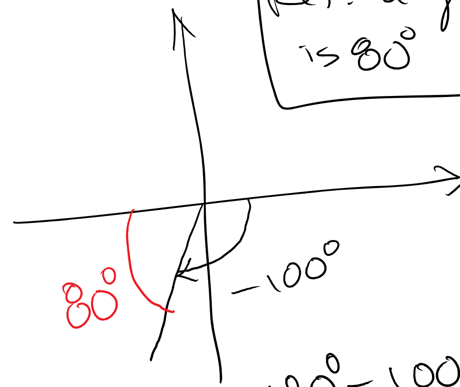


$197^\circ - 180^\circ = 17^\circ$

$\boxed{\text{Ref. angle is } 17^\circ}$

f) -100°

$\boxed{\text{Ref. angle is } 80^\circ}$



$180^\circ - 100^\circ = 80$

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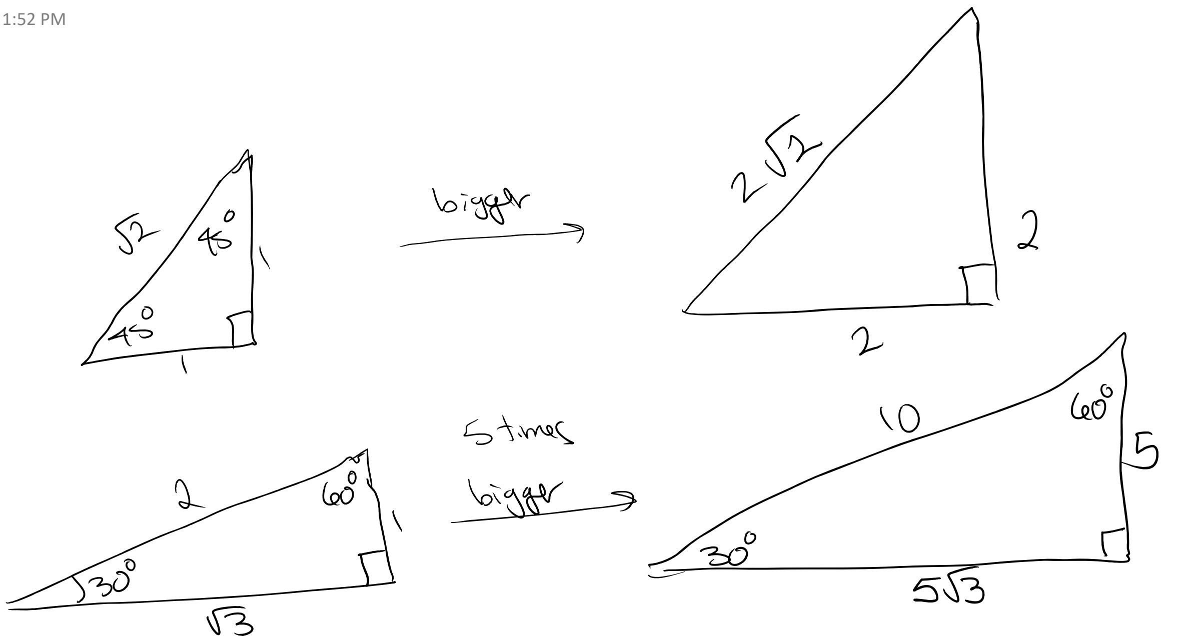
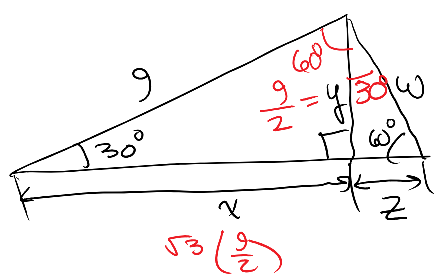
2.2.5



compare.
$2(?)=9$
multiply by $\frac{9}{2}=4.5$

2.1 # 13

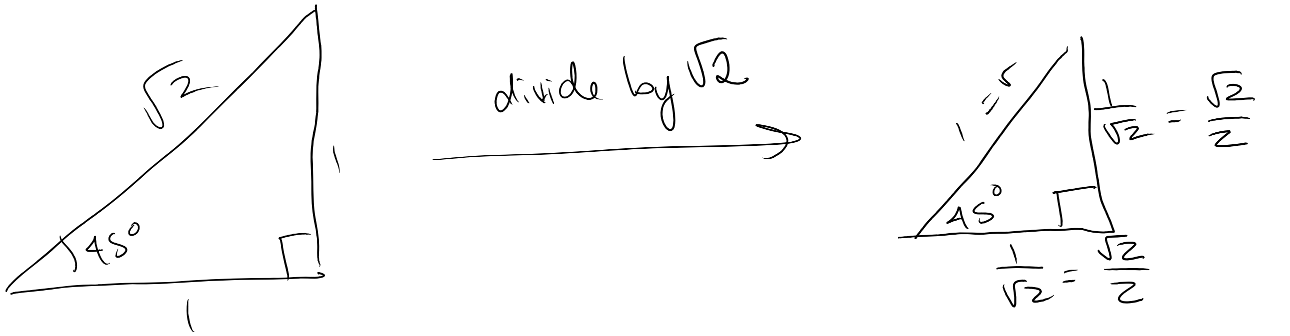


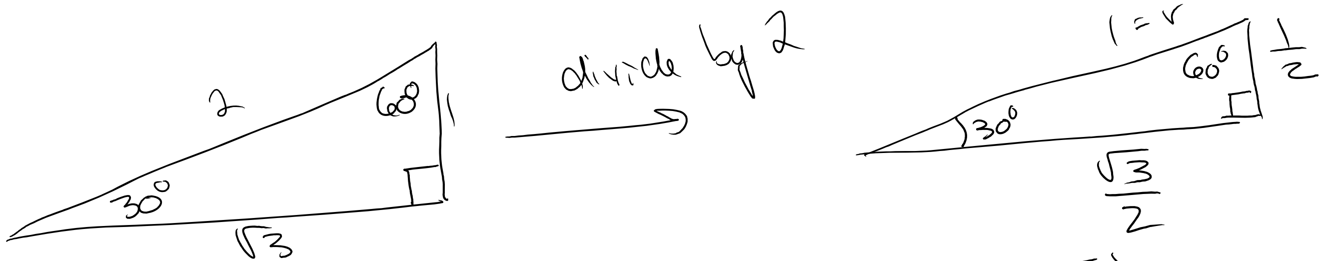
$$\frac{\frac{9}{2}}{\sqrt{3}} = \frac{z}{1}$$

or

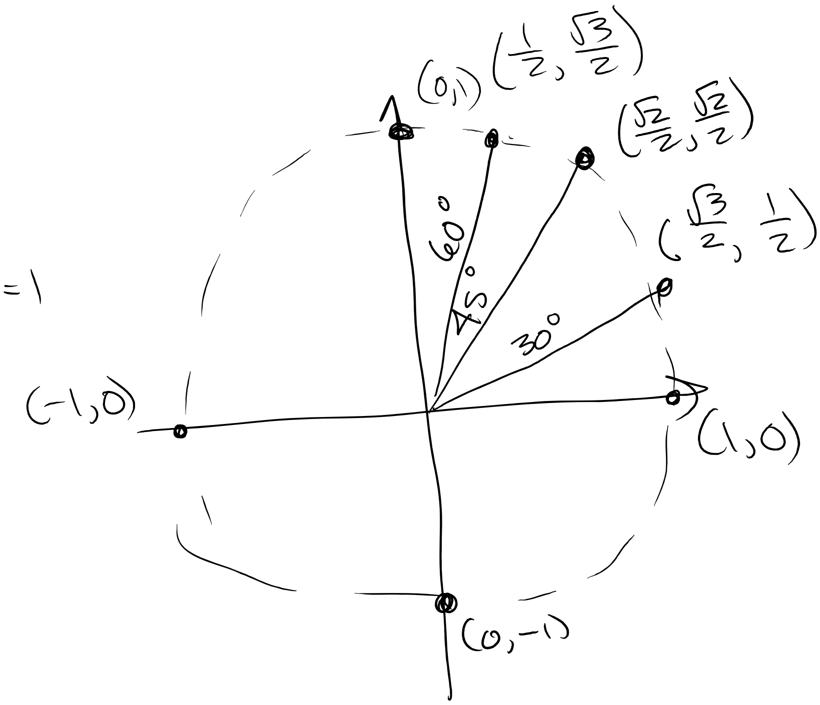
$$\tan 60^0 = \frac{\frac{9}{2}}{z} = \frac{\sqrt{3}}{1}$$

2.2.6



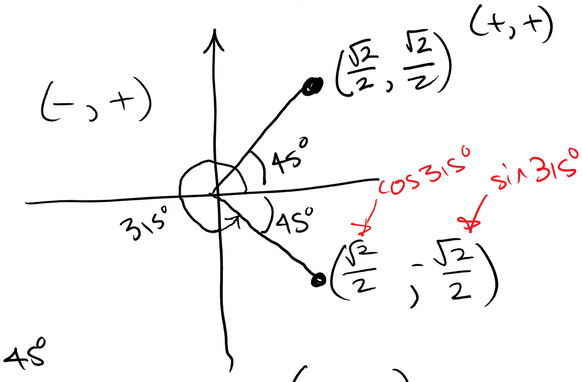


Learn these 3 points on unit circle $x^2+y^2=1$ $(r=1)$



2.2.7

Example 4: Find the values of the six trigonometric functions for 315°.



$360° - 315° = 45°$

sin 315° = $\boxed{-\frac{\sqrt{2}}{2}}$

cos 315° = $\boxed{\frac{\sqrt{2}}{2}}$

tan 315° = $\frac{\sin 315°}{\cos 315°} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = \boxed{-1}$

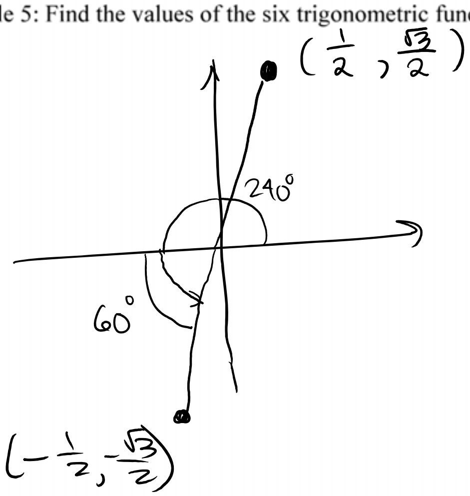
csc 315° = $-\frac{2}{\sqrt{2}} = -\frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \boxed{-\sqrt{2}}$

$\csc 315° = \frac{1}{\sin 315°}$

sec 315° = $\frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \boxed{\sqrt{2}}$

cot 315° = $\frac{\cos 315°}{\sin 315°} = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = \boxed{-1}$

Example 5: Find the values of the six trigonometric functions for ~~420~~ 240°.



$240° - 180° = 60°$

sin ~~420°~~ 240° = $\boxed{-\frac{\sqrt{3}}{2}}$

cos ~~420°~~ 240° = $\boxed{-\frac{1}{2}}$

tan ~~420°~~ 240° = $\frac{\sin 240°}{\cos 240°} = \frac{-\sqrt{3}/2}{-1/2} = \frac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$

csc ~~420°~~ 240° = $-\frac{2}{\sqrt{3}} = \boxed{-\frac{2\sqrt{3}}{3}}$

sec ~~420°~~ 240° = $-\frac{2}{1} = \boxed{-2}$

cot ~~420°~~ 240° = $\frac{1}{\sqrt{3}} = \boxed{\sqrt{3}}$

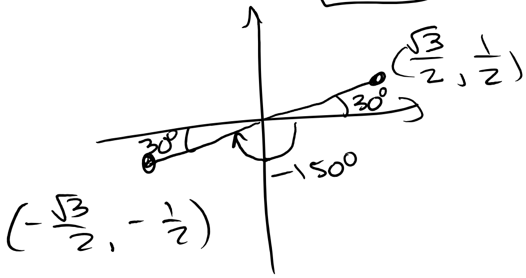
(reciprocal of tan 240°)

$\boxed{\text{Note: If } \frac{A/B}{C/B} = \frac{A}{C}}$

2.2.8

Example 6: Find the exact value of each expression.

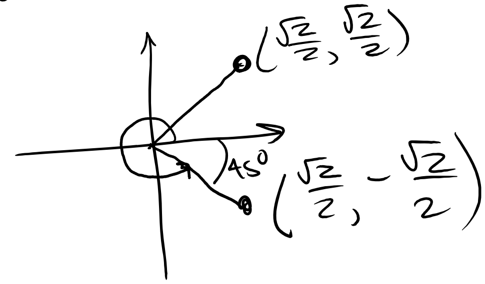
a) $\sin(-150°)$ $= \boxed{-\frac{1}{2}}$



b) $\cot 1035°$

$\frac{1035°}{360°} = 2.875$

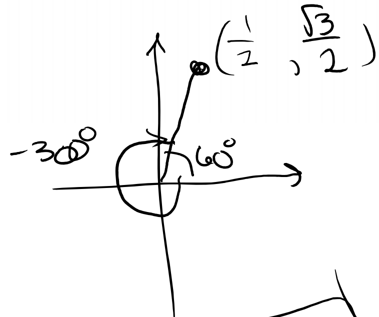
$1035° - 2(360°) = 315°$



$\cot 1035° = \frac{\cos 1035°}{\sin 1035°} = \frac{x}{y}$

$= \frac{\sqrt{2}/2}{-\sqrt{2}/2} = \boxed{-1}$

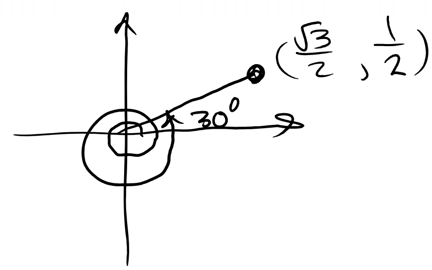
c) $\cos(-300°)$



$\cos(-300°) = \boxed{\frac{1}{2}}$

d) $\sec(750°)$

$2(360°) = 720°$



750° is coterminal with 30°

$\cos 750° = \frac{\sqrt{3}}{2}$

$\sec 750° = \frac{2}{\sqrt{3}}$

$= \boxed{\frac{2\sqrt{3}}{3}}$

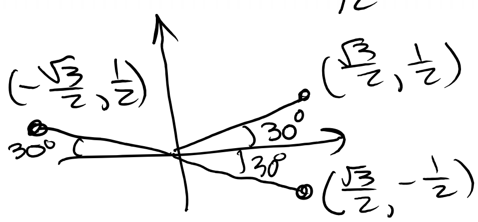
$\boxed{2.2.9}$

Example 7: Find all values of θ, if θ is in the interval $[0°, 360°)$ and has the given function value.

a) $\cot\theta = -\sqrt{3}$

$$\frac{\cos\theta}{\sin\theta} = \frac{-\sqrt{3}}{1} = -\frac{\sqrt{3}/2}{1/2}$$

$$= \frac{-\sqrt{3}/2}{1/2} = \frac{\sqrt{3}/2}{-1/2}$$

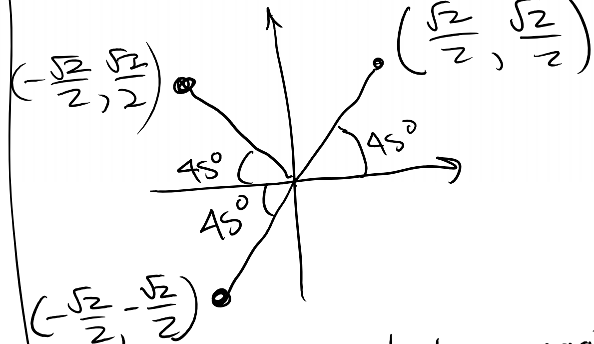


Q2: $180° - 30° = \boxed{150°}$

Q4: $360° - 30° = \boxed{330°}$

b) $\cos\theta = -\frac{\sqrt{2}}{2}$

cos θ ↓



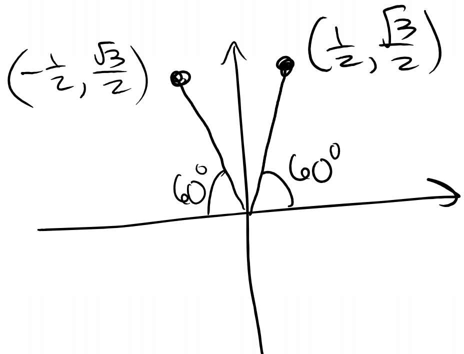
cos θ will be negative in QII and Q3.

In QII: $180° - 45° = \boxed{135°}$

In QIII, $180° + 45° = \boxed{225°}$

c) $\csc\theta = \frac{2\sqrt{3}}{3}$

$$\sin\theta = \frac{3}{2\sqrt{3}} \cdot \frac{\cancel{3}\sqrt{3}}{2\cdot\cancel{3}} = \frac{\sqrt{3}}{2}$$



In QI: $\boxed{60°}$

In QII: $180° - 60° = \boxed{120°}$

sin θ is pos. in QI and QII

Know the Reference angle families from 0° to 360°. The angles in each reference angle family will have the same function values, except for the signs. The sign of the values are based upon the quadrant of the angle.

Reference Angle	Reference angle family of angles	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
30°	30°, 150°, 210°, 330°	$\pm\frac{1}{2}$	$\pm\frac{\sqrt{3}}{2}$	$\pm\frac{\sqrt{3}}{3}$	$\pm\sqrt{3}$	$\pm\frac{2\sqrt{3}}{3}$	±2
45°	45°, 135°, 225°, 315°	$\pm\frac{\sqrt{2}}{2}$	$\pm\frac{\sqrt{2}}{2}$	±1	±1	$\pm\sqrt{2}$	$\pm\sqrt{2}$
60°	60°, 120°, 240°, 300°	$\pm\frac{\sqrt{3}}{2}$	$\pm\frac{1}{2}$	$\pm\sqrt{3}$	$\pm\frac{\sqrt{3}}{3}$	±2	$\pm\frac{2\sqrt{3}}{3}$