$\boxed{2.4.1}$

Recall: Inverse functions

Example:

x	$f(x) = x^3$
0	$0^3 = 0$
1	$1^3 = 1$
2	$2^3 = 8$
3	$3^3 = 27$
-1	$(-1)^3 = -1$
-2	$(-2)^3 = -8$

$\longrightarrow$

x	$f^{-1}(x) = \sqrt[3]{x}$
0	0
1	1
8	2
27	3
-1	-1
-8	-2

Ex:

x	$f(x) = x^2$
0	$0^2 = 0$
1	$1^2 = 1$
2	$2^2 = 4$
3	$3^2 = 9$
-1	$(-1)^2 = 1$
-2	$(-2)^2 = 4$
-3	$(-3)^2 = 9$

$\longrightarrow$

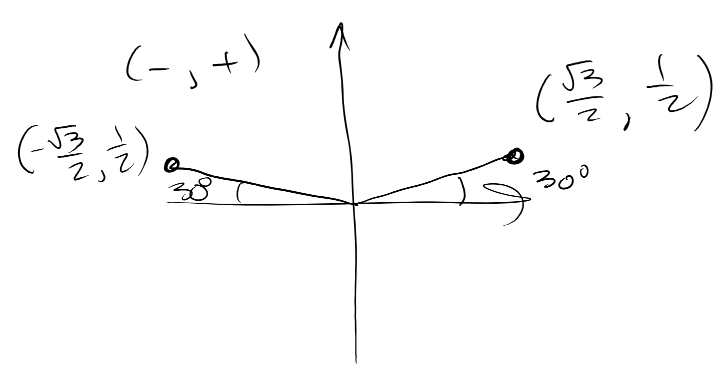
f^{-1} does not exist

x	
0	0
1	1
4	2
9	3
1	-1
4	-2

can't have 2 outputs

Ex:

x	$f(x) = \sin x$
30°	$\sin(30^\circ) = \frac{1}{2}$
150°	$\sin(150^\circ) = \frac{1}{2}$
390°	$\sin(390^\circ) = \frac{1}{2}$



$390°$ | $\sin(390°) = \frac{1}{2}$

$150° + 360° = 510°$ | $\sin(510°) = \frac{1}{2}$

so $f(x) = \sin x$ does not have an inverse.

However, the function $f(x) = \sin^{-1}(x)$ acts like an inverse function in that it gives you back angles when you put in a value.

Put in a real number between -1 and 1 $\Rightarrow$ $\sin^{-1}(x)$ gives you back an angle between $-90°$ and $90°$

Put in a real number between -1 and 1 $\Rightarrow$ $\cos^{-1}(x)$ gives you back an angle between $0°$ and $180°$.

Put in any real number $\Rightarrow$ $\tan^{-1}(x)$ gives you back an angle between $-90°$ and $90°$

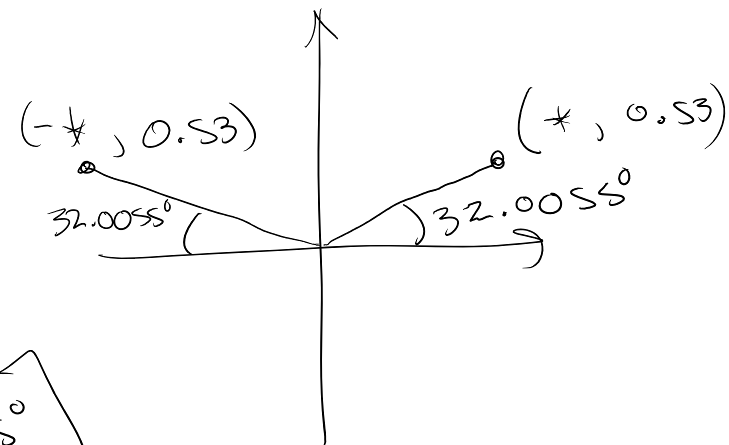
On calculator:

$\sin^{-1}(0.5) = 30°$

$\sin^{-1}(0.5) = 30^\circ$

Ex: solve $\sin\theta = 0.53$

$\sin^{-1}(0.53) \approx 32.0055^\circ$



QII: $180^\circ - 32.0055^\circ$

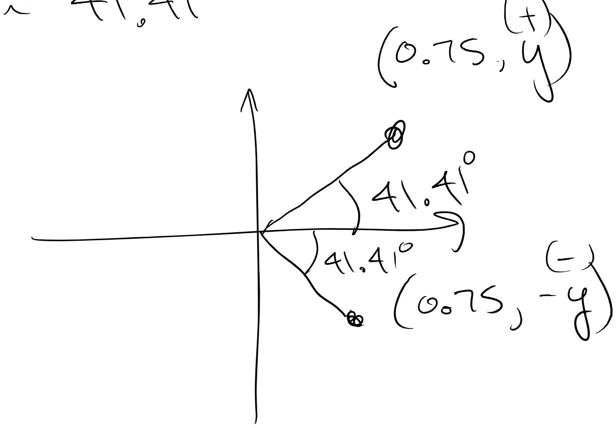
$\approx 147.9945^\circ$

$$\boxed{\theta \approx 32.0055^\circ, 147.9945^\circ}$$

Ex: For what values in $[0^\circ, 360^\circ)$ is

$\cos\theta = 0.75$?

Reference angle is $\cos^{-1}(0.75) \approx 41.41^\circ$



Q4: $360^\circ - 41.41^\circ$

$\approx 318.59^\circ$

$$\boxed{\theta \approx 41.41^\circ, 318.59^\circ}$$

Solving Right Triangles

Solving An Applied Trigonometry Problem

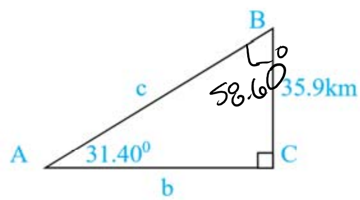
Step 1: Draw a sketch, and label it with the given information. Label the quantity to be found with a variable.
Step 2: Use the sketch to write an equation relating the given quantities to the variable.
Step 3: Solve the equation, and check that your answer makes sense.

Example 2:

To Solve a triangle means to find the missing sides and angles.

Solve each right triangle.

1.



$$31.40^\circ + 90^\circ + B = 180^\circ$$

$$31.40^\circ + B = 90^\circ$$

$$B = 90^\circ - 31.40^\circ = 58.6^\circ = B$$

$$\sin(31.40^\circ) = \frac{\text{opp}}{\text{hyp}} = \frac{35.9\text{ km}}{c}$$

$$c(\sin(31.40^\circ)) = 35.9\text{ km}$$

$$c = \frac{35.9\text{ km}}{\sin 31.40^\circ} \approx 68.9\text{ km}$$

OR

$$\tan(31.40^\circ) = \frac{\text{opp}}{\text{adj}} = \frac{35.9\text{ km}}{b}$$

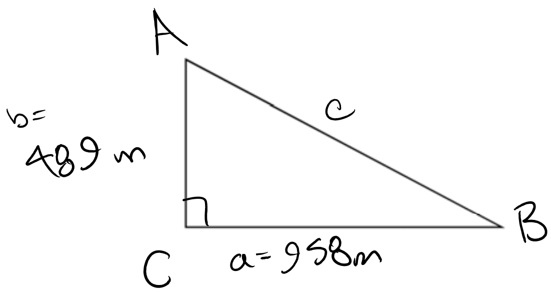
$$b\tan(31.40^\circ) = 35.9\text{ km}$$

$$b = \frac{35.9\text{ km}}{\tan(31.4^\circ)}$$

$$b = 58.8\text{ km}$$

$$\boxed{\begin{aligned} c &= 68.9\text{ km} \\ b &= 58.8\text{ km} \\ B &= 58.60^\circ \end{aligned}}$$

3. a = 958m, b = 489m



$$c^2 = a^2 + b^2$$

$$c^2 = (489\,m)^2 + (958\,m)^2$$

$$c^2 = 1\,156\,885\ m^2$$

$$c = \pm\sqrt{1\,156\,885}$$

$$\approx 1075.59\ m$$

$$\approx 1076\ m$$

$$\tan B = \frac{opp}{adj} = \frac{489\,m}{958\,m}$$

$$\tan B \approx 0.510438$$

$$B \approx \tan^{-1}(0.510438)$$

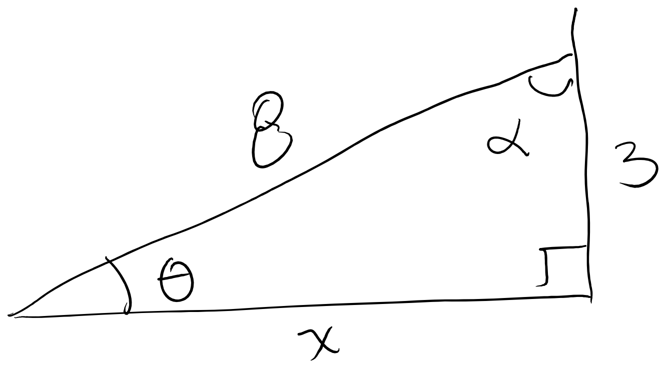
$$B \approx 27.0415^\circ$$

$$A \approx 90^\circ - 27.0415^\circ$$

$$\approx 62.95848763^\circ$$

$$\boxed{\begin{aligned} A &= 62.96^\circ \\ B &= 27.04^\circ \\ c &= 1076\ m \end{aligned}}$$

Example: Solve the triangle.



$\tan\theta = \frac{3}{x} = \frac{opp}{adj}$

$\cos\theta = \frac{adj}{hyp} = \frac{x}{8}$

$\sin\theta = \frac{opp}{hyp} = \frac{3}{8}$

$x^2 + 3^2 = 8^2$

$x^2 + 9 = 64$

$x^2 = 55$

$x = \pm\sqrt{55}$

$\boxed{x = \sqrt{55}}$

$\cos\theta = \frac{x}{8} = \frac{\sqrt{55}}{8}$

$\theta = \cos^{-1}\left(\frac{\sqrt{55}}{8}\right)$

$\theta \approx 22.02^\circ$

$\theta = \sin^{-1}(3/8)$

$\boxed{\theta \approx 22.02^\circ}$

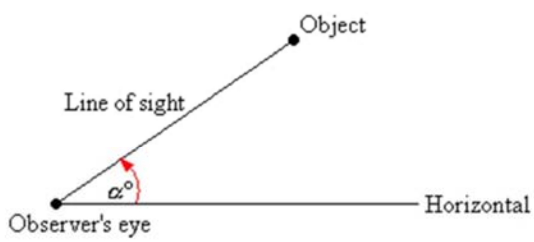
$\alpha = 90 - 22.02^\circ$

$\boxed{\alpha = 67.98^\circ}$

"α" is the Greek letter lower-case alpha

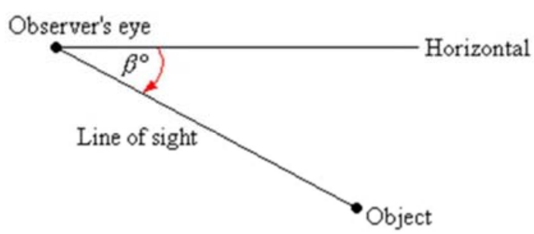
Angles of Elevation and Depression

The **angle of elevation** of an object as seen by an observer is the angle between the horizontal line and the line from the object to the observer's eye (the line of sight).



The angle of elevation of the object from the observer is $\alpha°$.

If the object is below the level of the observer, then the angle between the horizontal line and the observer's line of sight is called the **angle of depression**.



The angle of depression of the object from the observer is $\beta°$.

Example (in terms of bearing)

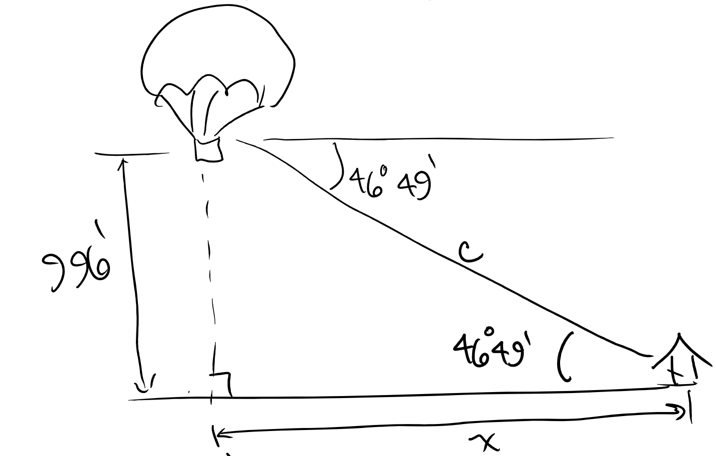
1. A fire is sighted due west of lookout A. The bearing of the fire from lookout B, 7.8 miles due south of A, is N 34°44'W. How far is the fire from B (to the nearest tenth of a mile)?

Answer in a sentence:

The headquarters is 935' away from the pt. below the balloon.

Examples (Angles of Elevation/Depression)

2. From a balloon 996 feet high, the angle of depression to the ranger headquarters is 46°49'. How far is the headquarters from a point on the ground directly below the balloon (to the nearest foot)?



$\sin(46°49') = \frac{996'}{c}$

we don't care about c

Instead, choose

$\tan(46°49') = \frac{opp}{adj} = \frac{996'}{x}$

$x \tan(46°49') = 996'$

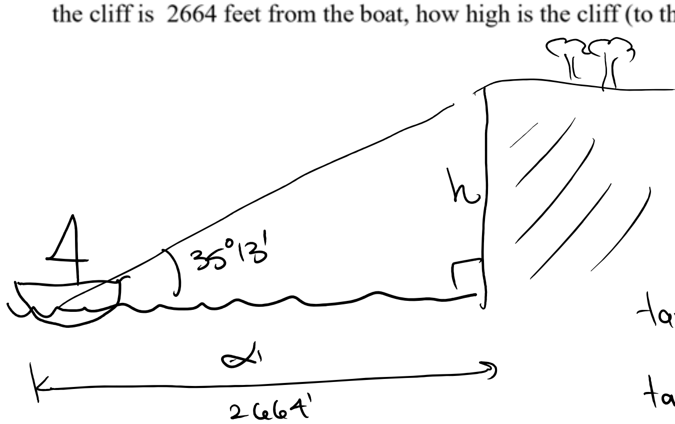
$x = \frac{996'}{\tan(46°49')} = \frac{996'}{\tan(46.8167°)}$

$x \approx 934.76' \approx 935'$

Change 46°49' to decimal degrees:

$46° + \frac{49}{60}° \approx 46.8167°$

3. From a boat on the lake, the angle of elevation to the top of a cliff is 35°13' If the base of the cliff is 2664 feet from the boat, how high is the cliff (to the nearest foot)?



$\tan(35°13') = \frac{opp}{adj}$

$\tan(35°13') = \frac{h}{2664'}$

$\tan\left(35° + \frac{13}{60}°\right) = \frac{h}{2664'}$

$\tan(35.2167°) = \frac{h}{2664'}$

$2664' \cdot \tan(35.2167°) = h$

$h \approx 1880.40606$

$h \approx 1880'$

The height of the cliff is 1880 ft.