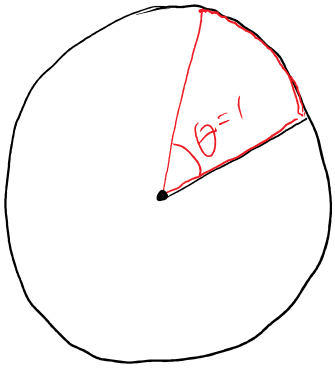
Thursday, February 21, 2019 9:09 AM

3.1: Radian Measure

Radian Measure: Suppose an angle θ has its vertex at the center of a circle. If θ intercepts an arc on the circle equal in length to the radius, then θ has a radian measure of 1.



π = ratio of circumference to diameter

$$\pi = \frac{C}{d}$$

$$\pi = \frac{C}{2r}$$

$$2r\pi = C$$

It turns out that the radian measure of 180° (half a circle) is π.

$\pi \approx 3.14159$

A whole circle: $\theta = 360°$, the intercepted arc is the circumference C

~~So $2\pi = 360°$~~

More correct way to say it: The angle 360° has radian measure of 2π.

Important: Radian measure is **unitless**.

Converting between degrees and radian measure: 3.1.2

* Multiply a degree measure by $\frac{\pi}{180^\circ}$ to get radian measure
* Multiply a radian measure by $\frac{180^\circ}{\pi}$ to get degree measure.

Ex: Convert 23 yards to inches.

$$23 \text{ yds}\left(\frac{3 \text{ ft}}{1 \text{ yd}}\right)\left(\frac{12 \text{ in}}{1 \text{ ft}}\right) = 828 \text{ in}$$

Ex: Convert 45° to radian measure.

$$45^\circ = 45^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{45\pi}{180} = \frac{5\pi}{20} = \boxed{\frac{\pi}{4}}$$

Note: no units!

Ex: convert -270° to radian measure.

$$-270^\circ = -270^\circ\left(\frac{\pi \text{ (radians)}}{180^\circ}\right) = \frac{-270\pi}{180} = -\frac{27\pi}{18}$$

$$= \boxed{-\frac{3\pi}{2}}$$

Ex: convert 139.7° to radian measure.

$$139.7^\circ = 139.7^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{139.7\pi}{180} = \frac{139.7\pi \cdot 10}{180 \cdot 10} = \boxed{\frac{1397\pi}{1800}}$$

exact answer

$$\approx \boxed{2.43822496}$$

3.1.3

Ex: Convert $\frac{9\pi}{4}$ to degrees.

$$\frac{9\pi}{4}(\text{radians})\left(\frac{180^\circ}{\pi(\text{radians})}\right) = \frac{9\cdot 180^\circ}{4} = \boxed{405^\circ}$$

Ex: convert $\frac{5\pi}{6}$ to degrees.

$$\frac{5\pi}{6}(\text{radians})\left(\frac{180^\circ}{\pi(\text{radians})}\right) = \frac{5\cdot 180^\circ}{6} = \boxed{150^\circ}$$

(30)

Ex: convert a radian measure of 4.25 to degrees.

$$4.25(\text{radians})\left(\frac{180^\circ}{\pi(\text{radians})}\right) = \frac{4.25(180)^\circ}{\pi}$$

$$\approx \boxed{243.51^\circ}$$

Writing our special angles $(30^\circ, 60^\circ, 45^\circ, 90^\circ)$ in radian measure.

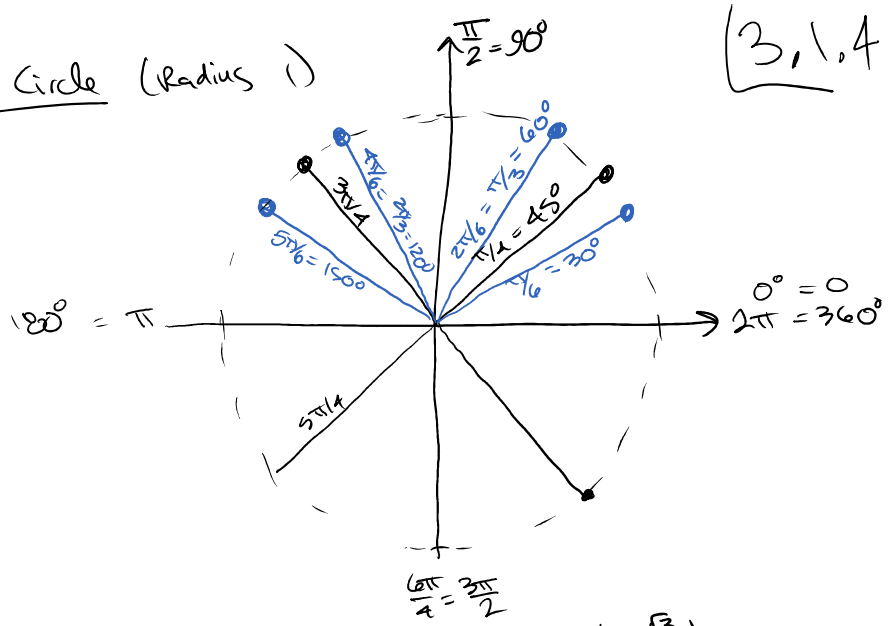
$30^\circ = 30^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{30\pi}{180} = \frac{\pi}{6}$ ← $2(30^\circ) = 2\left(\frac{\pi}{6}\right) = \frac{\pi}{3}$

$60^\circ = 60^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{60\pi}{180} = \frac{6\pi}{18} = \frac{\pi}{3}$

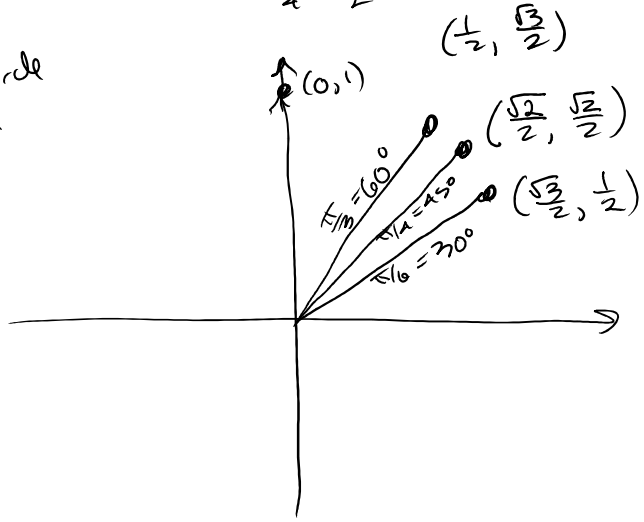
$45^\circ = \frac{\pi}{4}$, $90^\circ = \frac{\pi}{2}$

Unit Circle (radius 1)

3.1.4

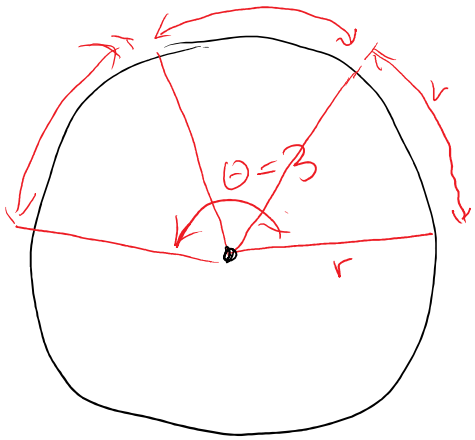


On unit circle
$x^2 + y^2 = 1$



3.1: Radian measure (cont'd) 3.1.5

Recall: Radian measure is the number of radii that lie along the arc intercepted by a central angle θ.



remember, radian measure is unitless.

Remember these:

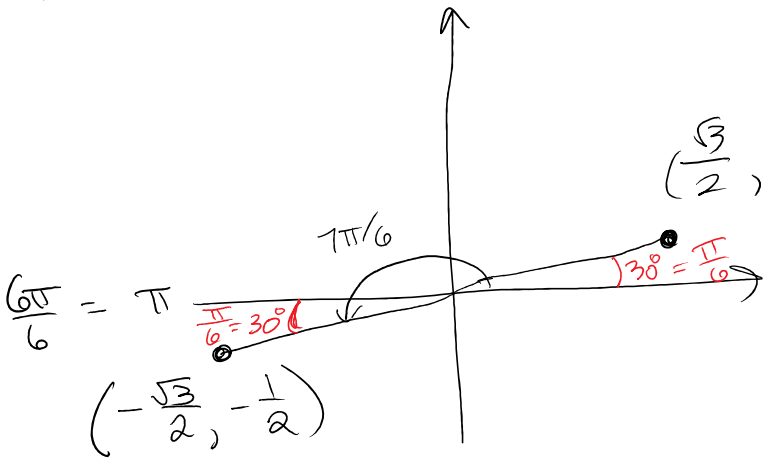
$180^\circ = \pi$ $\quad 360^\circ = 2\pi$

$90^\circ = \frac{\pi}{2}$ $\quad 30^\circ = \frac{\pi}{6}$

$45^\circ = \frac{\pi}{4}$ $\quad 60^\circ = \frac{\pi}{3} = \frac{2\pi}{6}$

Ex: Find $\cos \frac{7\pi}{6}$.

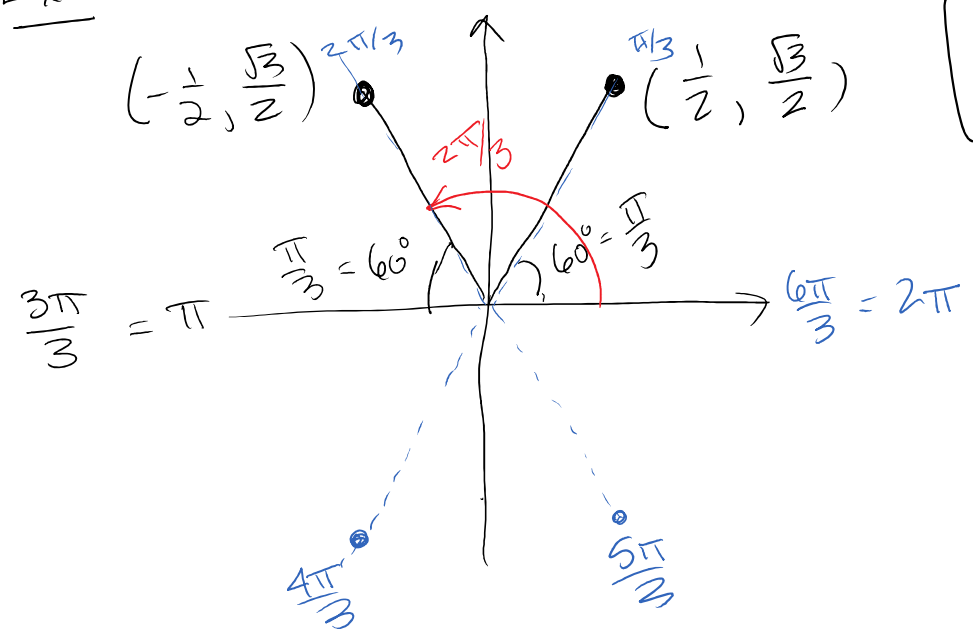
Reference angle: $\frac{\pi}{6} = 30^\circ$



$$\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

3.1.6

Ex: Find $\cos\left(\frac{2\pi}{3}\right)$ and $\tan\left(\frac{2\pi}{3}\right)$.



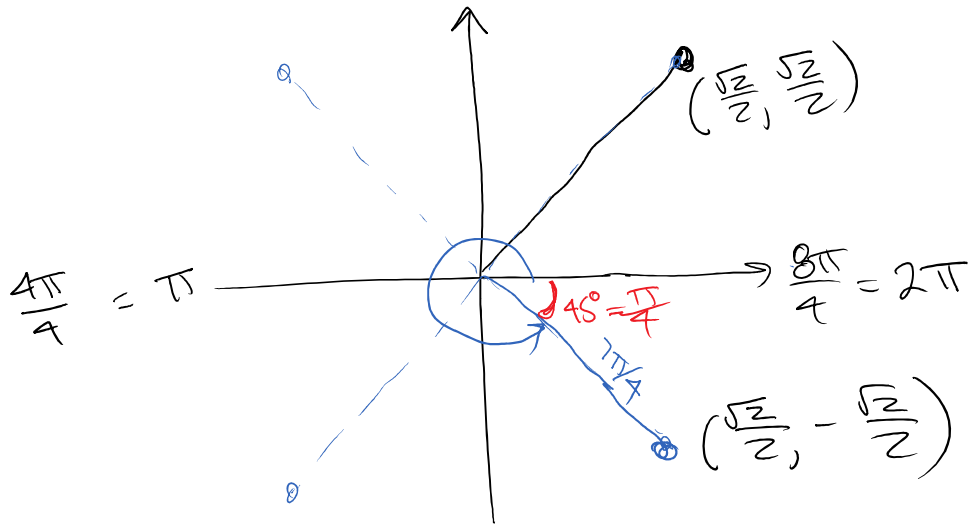
$$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$\tan\left(\frac{2\pi}{3}\right) = \frac{y}{x} = \frac{\sqrt{3}/2}{-1/2} = -\frac{\sqrt{3}}{1}$$

$$\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$$

Ex: Find $\sin\left(\frac{7\pi}{4}\right)$, $\csc\left(\frac{7\pi}{4}\right)$, $\tan\left(\frac{7\pi}{4}\right)$.

Ref. Angle: $\frac{\pi}{4}$



$$\sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$\csc\left(\frac{7\pi}{4}\right) = -\frac{2}{\sqrt{2}} = -\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$$

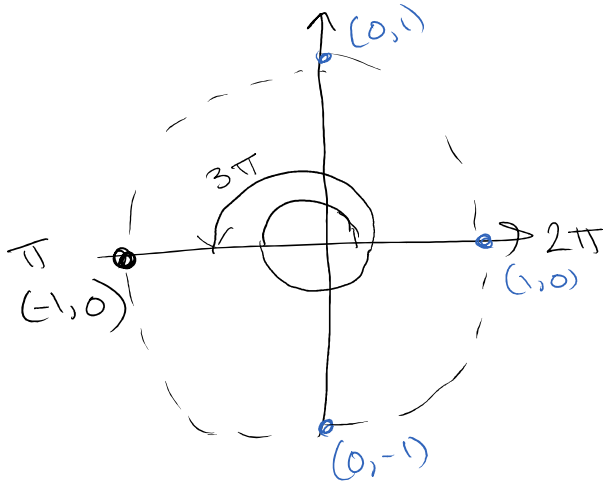
$$\csc\left(\frac{7\pi}{4}\right) = -\sqrt{2}$$

$$\tan\left(\frac{7\pi}{4}\right) = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$$

$$\tan\left(\frac{7\pi}{4}\right) = -1$$

3.1.7

Ex: Find $\cos(3\pi)$ and $\sin(3\pi)$.



$\cos(3\pi) = -1$

$\sin(3\pi) = 0$

$\tan(3\pi) = \frac{y}{x} = \frac{0}{-1} = 0$

$\cot(3\pi) = \frac{x}{y} = \frac{-1}{0}$ undefined

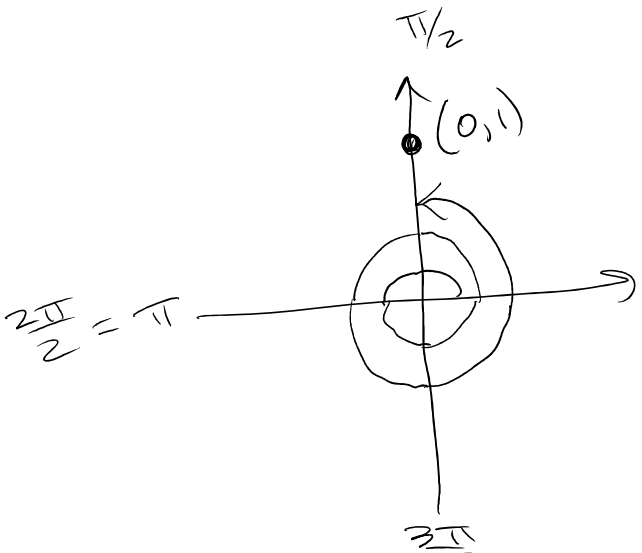
$\tan 3\pi = 0$

$\cot 3\pi$ is undefined

Ex: Find $\cos\left(\frac{9\pi}{2}\right)$ and $\tan\left(\frac{9\pi}{2}\right)$.

Note: $\frac{9\pi}{2} = \left(4\frac{1}{2}\right)\cdot\pi$

$= 4\pi + \frac{\pi}{2}$



$\cos\left(\frac{9\pi}{2}\right) = 0$

$\tan\left(\frac{9\pi}{2}\right) = \frac{y}{x} = \frac{1}{0}$ undef.

$\tan\left(\frac{9\pi}{2}\right)$ is undefined

Note: $\frac{9\pi}{2}(\text{rad}) \left/ \frac{180^\circ}{\pi(\text{rad})}\right.$

$= \frac{9\cdot 180^\circ}{2} = 9\cdot 90^\circ = 810^\circ$