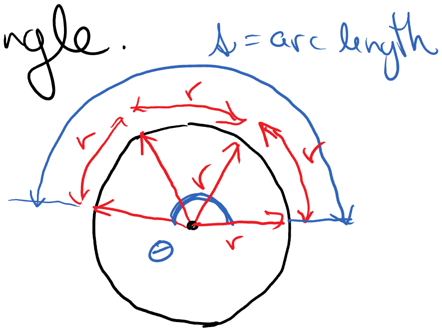
# 3.2: Applications of Radian Measure

## Arc Length:

Recall: The radian measure of an angle is the number of radii that fit along the arc of the circle intercepted (subtended) by that angle.




in this picture, $s = 3r$

Arc Length $s$ is given by

$$s = r\theta,$$

where $r$ is the radius of the circle and $\theta$ is the radian measure of the central angle.

Ex.: Find the arc length intercepted by an angle $\frac{2\pi}{5}$ on a circle of radius 3 cm.

$$s = r\theta = 3\text{cm}\left(\frac{2\pi}{5}\right) = \boxed{\frac{6\pi}{5}\text{ cm}} \approx \boxed{3.77\text{ cm}}$$

3.2.2

Ex: What arc length is intercepted by a $22^{\circ}$ angle on a circle of radius 5 miles?

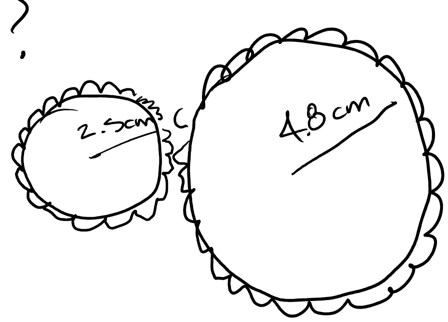
Change $\theta$ to radians:

$$22\cancel{^{\circ}}\left(\frac{\pi}{180\cancel{^{\circ}}}\right) = \frac{22\pi}{180}$$

$$s = r\theta = 5\text{ mi}\left(\frac{22\pi}{180}\right) \approx \boxed{1.92\text{ mi}}$$

$\boxed{3.2.3}$

Ex: Two gears are arranged so the smaller gear drives the larger one. The small gear has radius 2.5 cm and the large gear has radius 4.8 cm. If the small gear rotates through an angle of $225^\circ$, through how many degrees will the large gear rotate?




First, change $225^\circ$ to radian measure: $225^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{225\pi}{180}$

Find arc length covered by small gear:

$$s = r\theta = 2.5\text{cm}\left(\frac{225\pi}{180}\right) = 3.125\pi \text{ cm}$$

arc length on small gear must be equal to arc length on big gear.

$$s = r_{big}\theta_{big}$$

$$3.125\pi \text{ cm} = 4.8\text{cm}\,\theta_{big} \Rightarrow \theta_{big} = \frac{3.125\pi \text{ cm}}{4.8 \text{ cm}}$$

$$\approx 2.0453$$

(radians)

## 3.2.4

Change to degrees:

$$2.0453\ (\text{rad}\cancel{\text{ians}})\left(\frac{180^\circ}{\pi\ (\text{rad}\cancel{\text{ians}})}\right)$$

$$\approx 117.1875^\circ$$

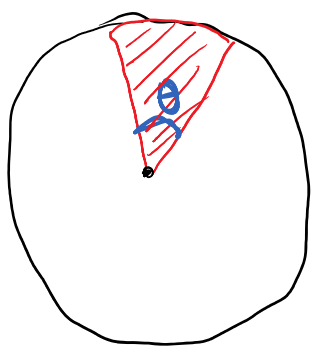
$$\approx \boxed{117^\circ}$$

Note: There is a latitude/longitude problem in my 3.2 notes from 8:00 MW class

3.2.5

## Sector Area:

Suppose a central angle $\theta$ cuts off a sector of a circle. What is the sector area?




Area of whole circle is $\pi r^2$.

Central angle for whole circle is $2\pi$

$$\frac{\text{Sector area}}{\pi r^2} = \frac{\theta}{2\pi}$$

$$\text{Sector area} = \frac{\theta}{2\cancel{\pi}} \cdot \cancel{\pi} r^2$$

$$\text{Sector area} = \frac{\theta r^2}{2}$$

$$\boxed{\text{Sector Area} = \frac{1}{2} r^2 \theta}$$

where $r$ is the radius and $\theta$ is the radian measure of the central angle.

Ex: What is the area of a sector cut off by a central angle of $130^\circ$? Radius is 10 ft.

Could change $130^\circ$ to radian measure and use $A = \frac{1}{2} r^2 \theta$.

Or, use a proportion:

$$\frac{\text{Sector area}}{\pi r^2} = \frac{13\cancel{0}^\circ}{36\cancel{0}^\circ}$$

$$\text{Sector area} = \frac{13\cancel{0}}{36\cancel{0}} \cdot \pi (10 \text{ ft})^2$$

$$= \frac{13}{36} \cdot \pi (100 \text{ ft}^2) = \boxed{\frac{1300\pi}{36} \text{ ft}^2}$$