4.1: Graphs of Sine and Cosine Functions

We will now think of our trig functions as functions of real numbers. The input of the function is the radian measure of an angle.

$$f(x) = \sin(x)$$

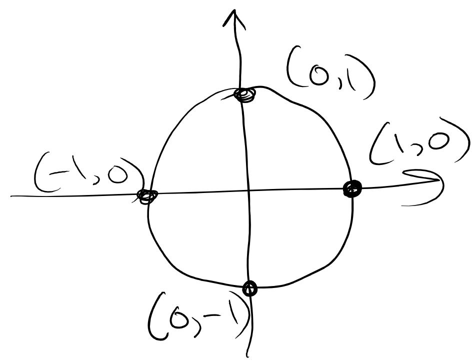
$$y = \sin(x)$$

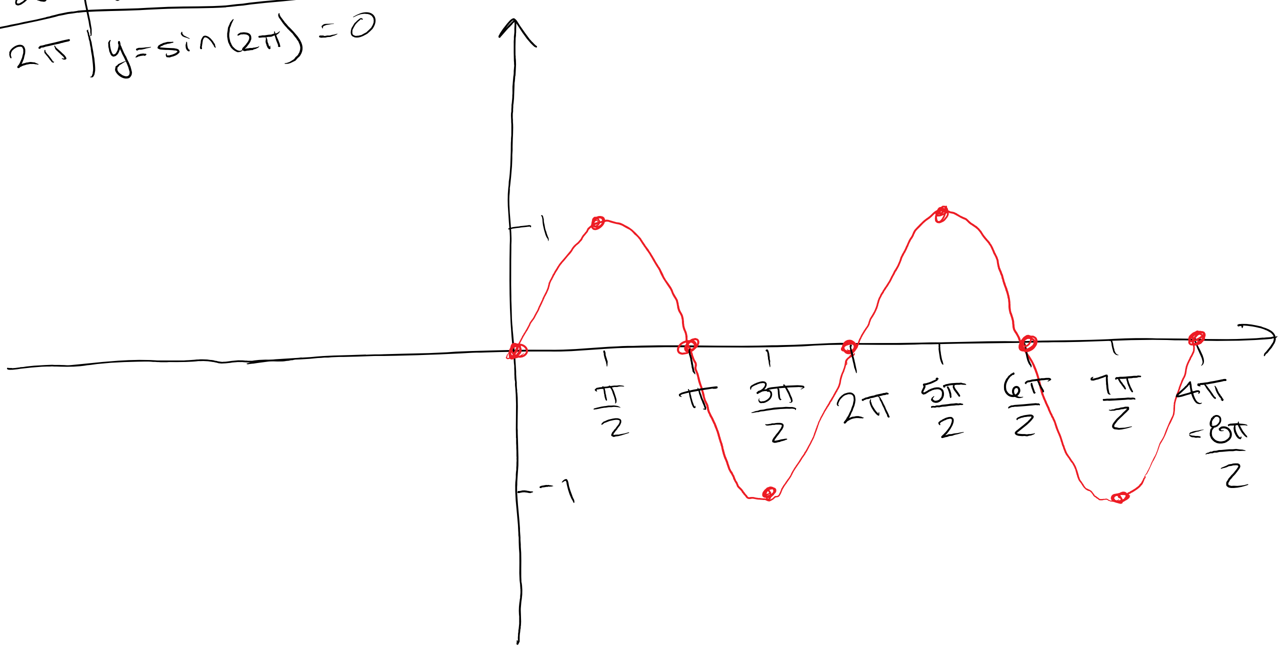
4.1.2

The trig functions are periodic. They repeat.

$y = \sin x$ repeats itself every 2π. On the interval $[0, 2\pi)$, we get:

x	$f(x) = \sin x$
0	$y = \sin(0) = 0$
$\frac{\pi}{2}$	$y = \sin\left(\frac{\pi}{2}\right) = 1$
π	$y = \sin(\pi) = 0$
$\frac{3\pi}{2}$	$y = \sin\left(\frac{3\pi}{2}\right) = -1$
2π	$y = \sin(2\pi) = 0$

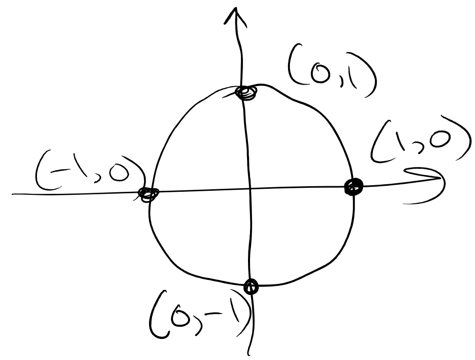


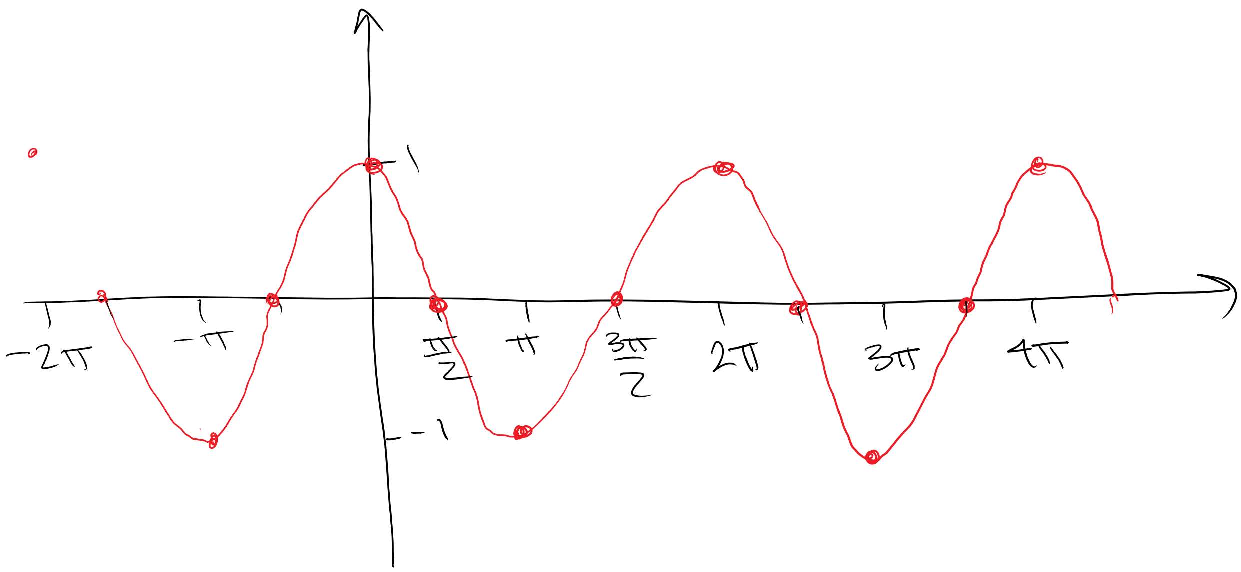


Graph of $f(x) = \cos x$

4.1.3

x	$f(x) = \cos x$
0	$y = \cos(0) = 1$
$\pi/2$	$y = \cos\left(\frac{\pi}{2}\right) = 0$
π	$y = \cos(\pi) = -1$
$3\pi/2$	$y = \cos\left(\frac{3\pi}{2}\right) = 0$
2π	$y = \cos(2\pi) = 1$





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4.1: Graphs of Sine and Cosine (cont'd) 4.1.4

If a periodic function has a maximum and minimum y-value, then the amplitude of the function is

$$\text{Amplitude} = \frac{1}{2}(y_{max} - y_{min})$$

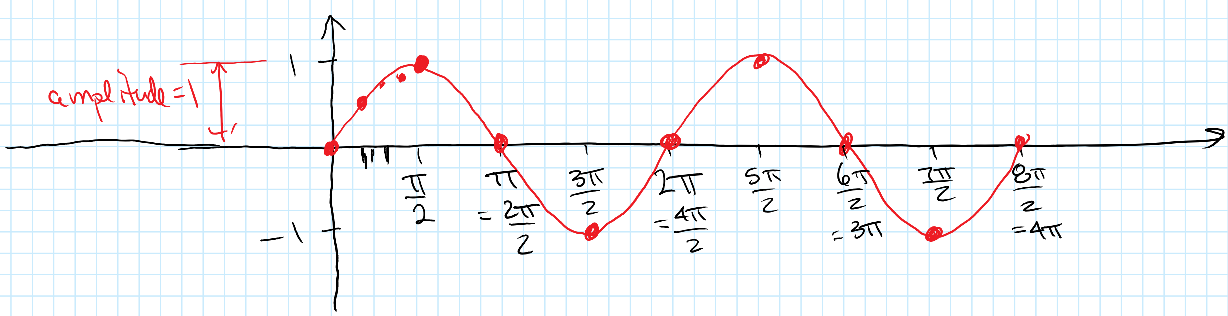
Ex: What is the amplitude of $f(x) = \sin x$?

maximum y-value: (max value of $f(x)$): 1

minimum y-value (min value of $f(x)$): -1

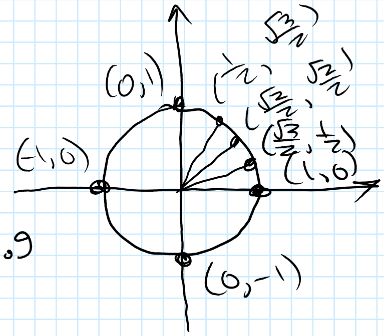
$$\text{Amplitude} = \frac{1}{2}(y_{max} - y_{min})$$
$$= \frac{1}{2}(1-(-1)) = \frac{1}{2}(1+1) = \frac{1}{2}(2) = 1.$$

For sin and cos, the amplitude is the vertical distance between the midline of the graph and the max.

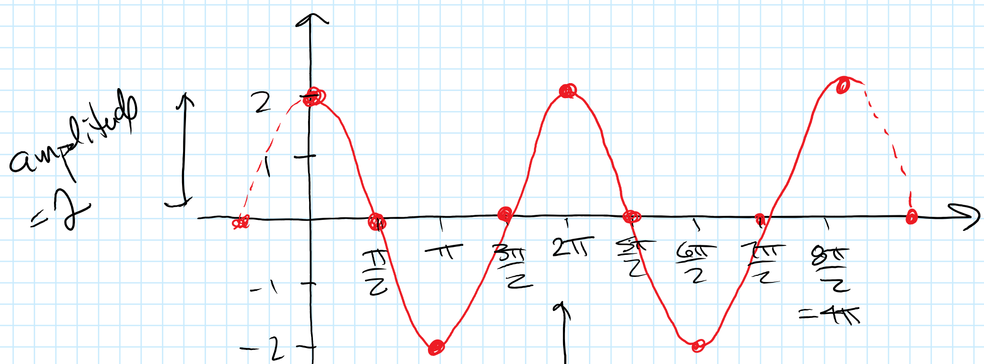


The period of a periodic function is the length of one cycle. The period of $y = \sin x$ and $y = \cos x$ is 2π.

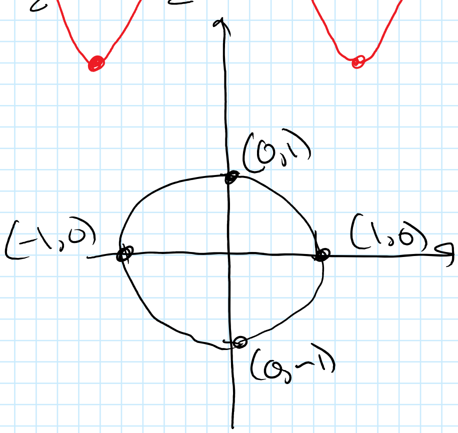
x	$f(x)$
0	$y = \sin(0) = 0$
$\frac{\pi}{6}$	$y = \sin(\frac{\pi}{6}) = \frac{1}{2} = 0.5$
$\frac{\pi}{4}$	$y = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} \approx 0.707 \approx 0.7$
$\frac{\pi}{3}$	$y = \sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2} \approx 0.866 \approx 0.9$
$\frac{\pi}{2}$	$y = \sin(\frac{\pi}{2}) = 1$
π	$y = \sin(\pi) = 0$
$\frac{3\pi}{2}$	$y = \sin(\frac{3\pi}{2}) = -1$
2π	$y = \sin(2\pi) = 0$



Ex: Sketch the graph of $f(x) = 2\cos(x)$ (two periods)



x	$y = 2\cos(x)$
0	$y = 2\cos(0) = 2(1) = 2$
$\frac{\pi}{2}$	$y = 2\cos(\frac{\pi}{2}) = 2(0) = 0$
π	$y = 2\cos(\pi) = 2(-1) = -2$
$\frac{3\pi}{2}$	$y = 2\cos(\frac{3\pi}{2}) = 2(0) = 0$
2π	$y = 2\cos(2\pi) = 2(1) = 2$



$\boxed{\text{amplitude} = 2}$

4.1.6

Period: The length of one cycle.

The period of a periodic function f is the minimum value of p such that $f(x+p) = f(x)$ for every x.

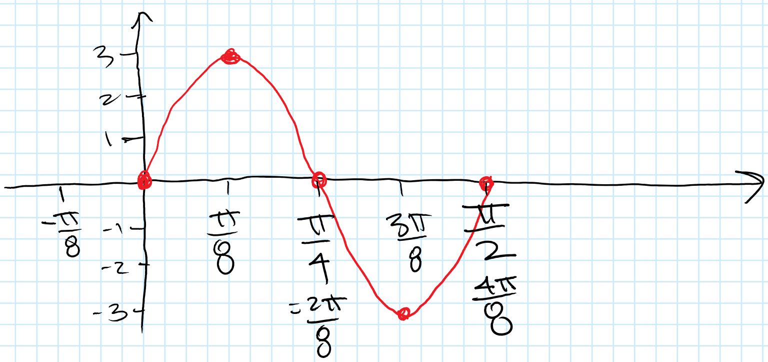
Ex: Graph $f(x) = 3\sin(4x)$.

For $y = \sin(x)$, or $y = 3\sin x$, one period is $0 \le x < 2\pi$.

For $y = \sin(4x)$, or $y = 3\sin(4x)$, one period is $0 \le 4x < 2\pi$

$$\frac{0}{4} \le \frac{4x}{4} < \frac{2\pi}{4}$$

$$0 \le x < \frac{\pi}{2}$$



$\boxed{\text{Period} = \frac{\pi}{2}}$

$\boxed{\text{Amplitude} = 3}$

4.1 #7

Ex: Graph two periods of $y = -4\cos(3x)$.

Amplitude: $|-4| = 4$

Period of $y = \cos(x)$ is $0 \le x < 2\pi$

(or $y = 4\cos x$ or $y = -4\cos x$)

Period of $y = -4\cos(3x)$ is $0 \le 3x < 2\pi$

(or $y = \cos(3x)$)

$\frac{0}{3} \le \frac{3x}{3} < \frac{2\pi}{3}$

$0 \le x < \frac{2\pi}{3}$

Note: For $y = A\cos(Bx)$
or $y = A\sin(Bx)$,
the amplitude is $|A|$.
The period is $\frac{2\pi}{B}$

$y = -4\cos(3x)$

Period $= \frac{2\pi}{3}$