

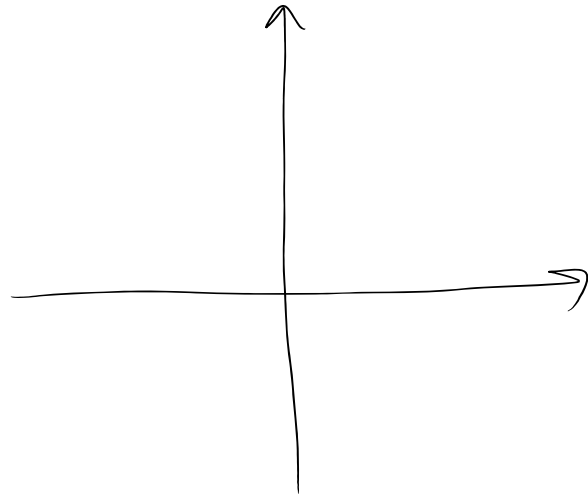
4.2.1

4.2: Translations of the Sine and Cosine Functions

Recall: Example: Graph $f(x) = x^2 + 3$.

Basic Function: $y = x^2$

x	$y = x^2$
-3	$(-3)^2 = 9$
-2	$(-2)^2 = 4$
-1	1
0	0
1	1
2	4
3	9

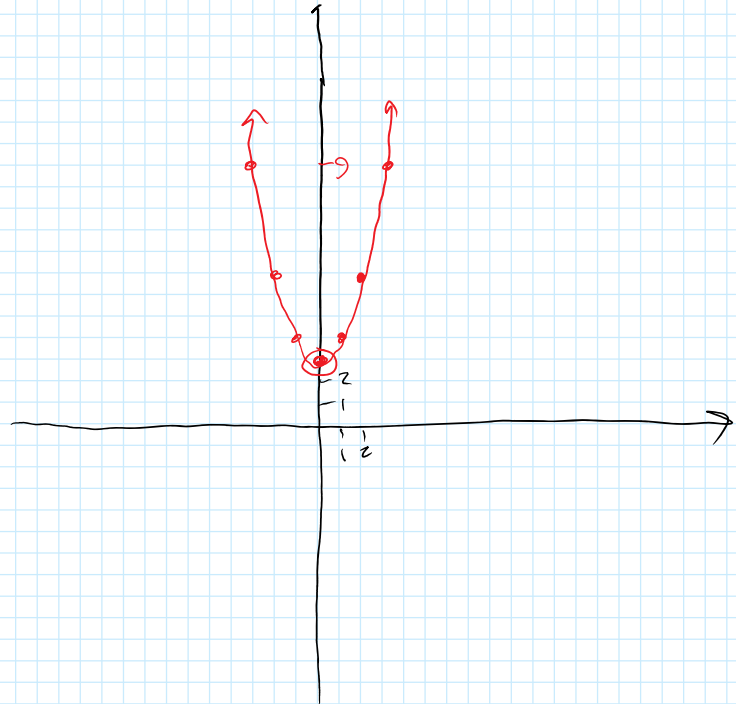


See next page

A.2.2

$$y = x^2 + 3$$

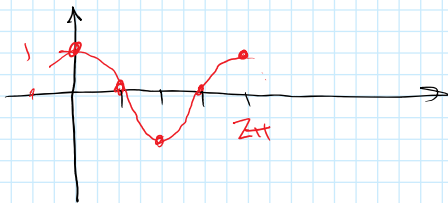
Adding $+3$ will shift the graph up 3.



A horizontal or vertical shift is called a translation.

Ex: Graph $y = -4 + \cos(x)$ over two periods.

Basic shape of $y = \cos x$:



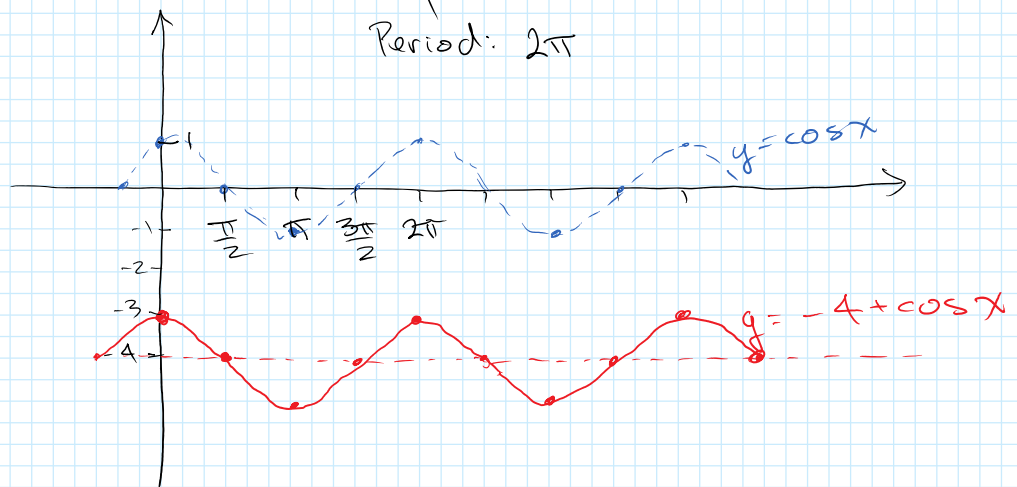
x	$y = \cos x$
0	$\cos(0) = 1$
$\frac{\pi}{2}$	$\cos(\frac{\pi}{2}) = 0$
π	$\cos(\pi) = -1$
$\frac{3\pi}{2}$	$\cos(\frac{3\pi}{2}) = 0$
2π	$\cos(2\pi) = 1$

Note: $y = -4 + \cos(x)$ is equivalent to $y = \cos x - 4$

Vertical shift: down 4

Amplitude: 1

Period: 2π



4.2.3

Ex: Graph $y = 3 - 2\sin(4x)$

Basic function: $y = \sin x$

Period: For $y = \sin(x)$, 1 period is $0 \leq x \leq 2\pi$

For $y = \sin(4x)$, 1 period is $0 \leq 4x \leq 2\pi$

$$\frac{0}{4} \leq \frac{4x}{4} \leq \frac{2\pi}{4}$$

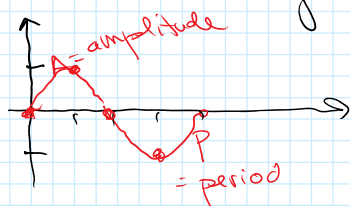
$$0 \leq x \leq \frac{\pi}{2}$$

Period: $\frac{\pi}{2}$

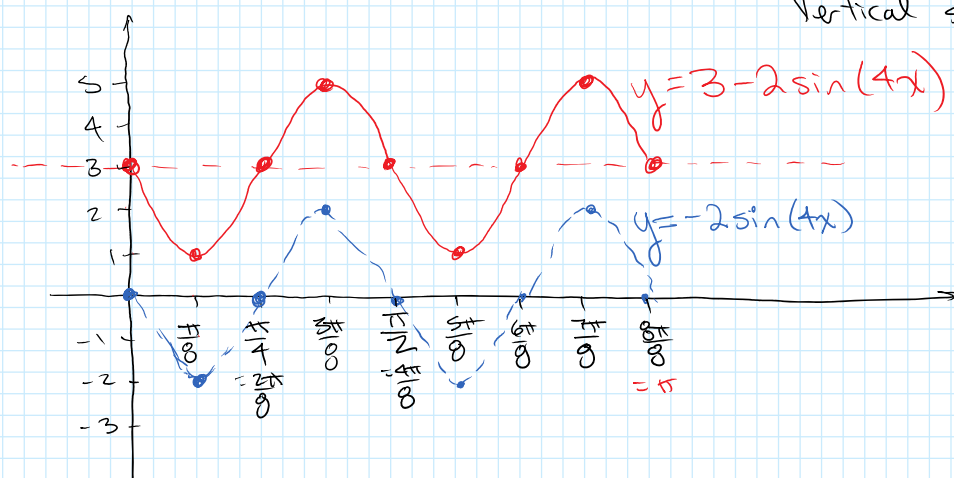
Amplitude: $A = |-2| = 2$

Reflection about x -axis before shifting

Vertical shift: up 3



$$y = 3 - 2\sin(4x)$$



4.2.4

Horizontal Shifts: Graphing $y = \sin(x-d)$ or $y = \cos(x-d)$

Recall: To graph $y = (x-h)^2$, start with $y = x^2$ and shift it right h ,
(for $h > 0$)

To graph $y = (x+h)^2$, start with $y = x^2$ and shift it left h
($h > 0$)

Ex.: Graph $y = \sin(x - \frac{\pi}{4})$ over 1 period.

Amplitude: 1

Period: 2π

Vertical Shift: none

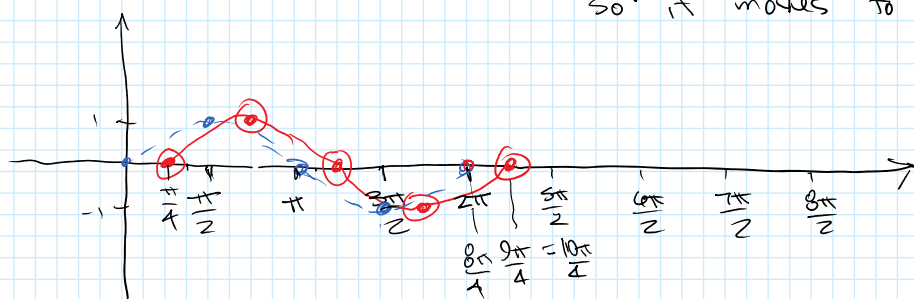
Horizontal Shift (Phase Shift): $\frac{\pi}{4}$ to the right

The horizontal shift is called a phase shift for periodic functions.

Reflection: none

Note: $x - \frac{\pi}{4} = 0$

$x = \frac{\pi}{4}$ (starting value for basic period)
so it moves to the right π



Note: $0 + \frac{\pi}{4} = \frac{\pi}{4}$
 $\frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$
 $\pi + \frac{\pi}{4} = \frac{5\pi}{4}$
 $\frac{3\pi}{2} + \frac{\pi}{4} = \frac{7\pi}{4}$
 $2\pi + \frac{\pi}{4} = \frac{9\pi}{4}$

Ex.: Graph $y = 5 \cos(2x + \frac{2\pi}{3})$

Rewrite: $y = 5 \cos(2(x + \frac{\pi}{3}))$

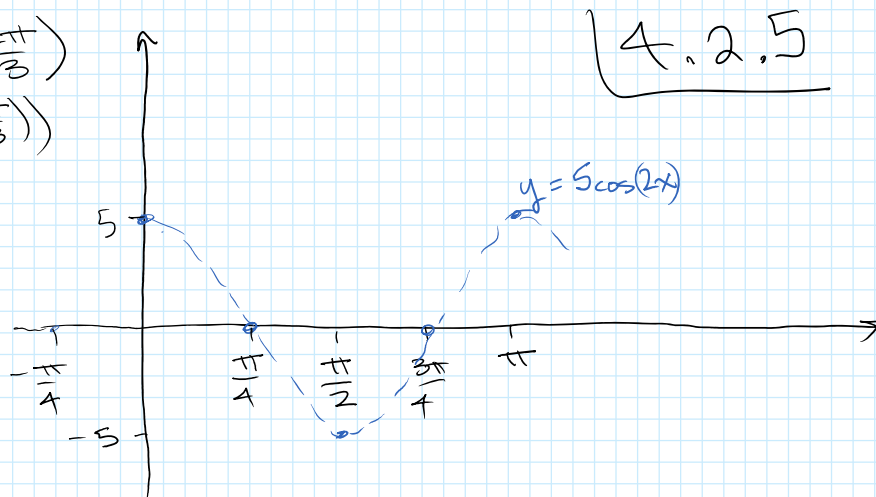
Amplitude: 5

Period: $\frac{2\pi}{2} = \pi$

Phase shift: $\frac{\pi}{3}$ to the left

Vertical shift: none

Reflection: none



$$0 - \frac{\pi}{3} = -\frac{\pi}{3}$$

$$\frac{\pi}{4} - \frac{\pi}{3} = \frac{3\pi}{12} - \frac{4\pi}{12} = -\frac{\pi}{12}$$

$$\frac{\pi}{2} - \frac{\pi}{3} = \frac{3\pi}{6} - \frac{2\pi}{6} = \frac{\pi}{6}$$

Tip: Write the phase shift and the measurement for 1 tick mark with the same denominator.

$$1^{\text{st}} \text{ tick mark: } \frac{\pi}{4} = \frac{3\pi}{12}$$

$$\text{phase shift: left } \frac{\pi}{3} \Rightarrow \text{left } \frac{4\pi}{12}$$

