5.1.1

5.1: Fundamental Identities

Reciprocal Identities

$\cot\theta = \frac{1}{\tan\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\csc\theta = \frac{1}{\sin\theta}$

Note: also $\tan\theta = \frac{1}{\cot\theta}$

$\cos\theta = \frac{1}{\sec\theta}$

$\sin\theta = \frac{1}{\csc\theta}$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$1 + \cot^2\theta = \csc^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

Opposite-angle identities

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

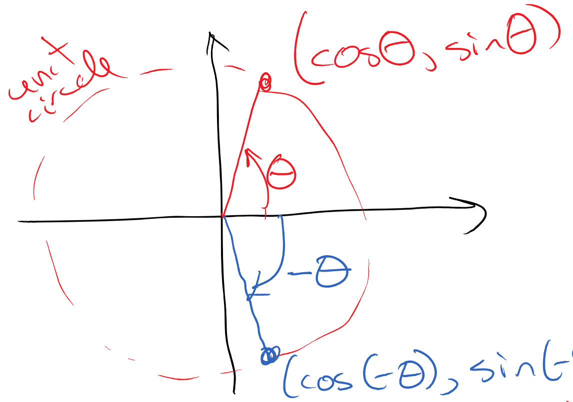
$\csc(-\theta) = -\csc\theta$

$\sec(-\theta) = \sec\theta$

$\cot(-\theta) = -\cot\theta$

5.1.2

Why are the opposite-angle identities true?



$(\cos(-\theta), \sin(-\theta))$ must be the same!

must be same as $(\cos\theta, -\sin\theta)$

so then $\tan(-\theta) = \dfrac{\sin(-\theta)}{\cos(-\theta)} = \dfrac{-\sin\theta}{\cos\theta} = -\tan\theta$

Pythagorean Identities:

We can use the 1st one to get 2 more.

$$\sin^2\theta + \cos^2\theta = 1$$

Divide by $\sin^2\theta$:

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

Start over and divide by $\cos^2\theta$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

Note: Why $\dfrac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$?

$$\frac{\cos^2\theta}{\sin^2\theta} = \left(\frac{\cos\theta}{\sin\theta}\right)^2 = (\cot\theta)^2$$

$$= \cot^2\theta$$

Ex: Suppose $\sin\theta = -\frac{2}{9}$. What is $\sin(-\theta)$?

$$\boxed{\sin(-\theta) = \frac{2}{9}}$$

$$\sin(-\theta) = -\sin\theta = -\left(-\frac{2}{9}\right) = \frac{2}{9}$$

Ex: Suppose $\cos(\theta) = 0.37$. What is $\cos(-\theta)$?

$$\boxed{\cos(-\theta) = 0.37}$$

Ex: If $\csc\beta = -8$, then what is $\sin(-\beta)$?

$\csc\beta = -8$

so $\sin\beta = -\frac{1}{8}$

$$\boxed{\sin(-\beta) = \frac{1}{8}}$$

5.1.4

Ex: Given that $\tan\theta = -\frac{1}{4}$ and θ is in Quadrant IV, find the remaining trig functions.

$$\boxed{\begin{aligned} \tan\theta &= -\frac{1}{4} \\ \cot\theta &= -\frac{4}{1} = -4 \\ \sin\theta &= -\frac{1}{\sqrt{17}} \\ \cos\theta &= \frac{4}{\sqrt{17}} \\ \csc\theta &= -\sqrt{17} \\ \sec\theta &= \frac{\sqrt{17}}{4} \end{aligned}}$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\left(-\frac{1}{4}\right)^2 + 1 = \sec^2\theta$$

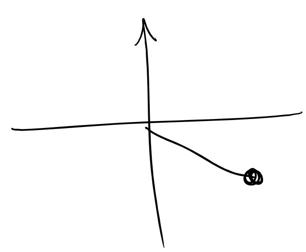
$$\frac{1}{16} + 1 = \sec^2\theta$$

$$\frac{1}{16} + \frac{16}{16} = \sec^2\theta$$

$$\frac{17}{16} = \sec^2\theta$$

$$\sec\theta = \pm\sqrt{\frac{17}{16}} = \pm\frac{\sqrt{17}}{4}$$

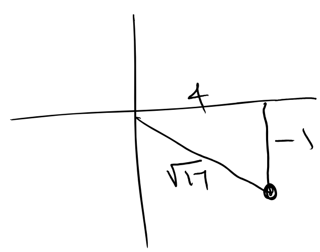
$$\cos\theta = \pm\frac{4}{\sqrt{17}}$$ still need to decide plus or minus.



Quadrant 4, so $\cos\theta$ is positive

$$\cos\theta = +\frac{4}{\sqrt{17}}$$

$$\sec\theta = +\frac{\sqrt{17}}{4}$$



$$\cos^2\theta + \sin^2\theta = 1$$

$$\left(\frac{4}{\sqrt{17}}\right)^2 + \sin^2\theta = 1$$

$$\frac{16}{17} + \sin^2\theta = 1$$

$$\sin^2\theta = 1 - \frac{16}{17}$$

$= \frac{17}{17} - \frac{16}{17}$

5.1.5

$\sin^2\theta = \frac{1}{17}$

$\sin\theta = \pm\sqrt{\frac{1}{17}} = \pm\frac{1}{\sqrt{17}}$. Quadrant 4, so $\sin\theta$ is negative.

$\sin\theta = -\frac{1}{\sqrt{17}}$

$\csc\theta = -\frac{\sqrt{17}}{1} = -\sqrt{17}$

simplify and write with no quotients.

Ex: $\cot\theta \sin\theta$

$\cot\theta \sin\theta$

$= \frac{\cos\theta}{\sin\theta} \cdot \sin\theta = \frac{\cos\theta}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{1} = \frac{\cos\theta}{1} = \boxed{\cos\theta}$

5.1.6

Ex: $(1-\cos\theta)(1+\sec\theta)$

$= 1 + \sec\theta - \cos\theta - \cos\theta\sec\theta$

$= 1 + \frac{1}{\cos\theta} - \cos\theta - \cos\theta\left(\frac{1}{\cos\theta}\right)$

$= 1 + \frac{1}{\cos\theta} - \cos\theta - 1$

$= \frac{1}{\cos\theta} - \cos\theta$

$= \boxed{\sec\theta - \cos\theta}$

Ex: $\frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$

$= \frac{\cos^2\theta}{\sin\theta\cos\theta} - \frac{\sin^2\theta}{\sin\theta\cos\theta}$

$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$

$= \boxed{\cot\theta - \tan\theta}$

Note: $\frac{2}{7} + \frac{3}{7}$

$= \frac{2+3}{7} = \frac{5}{7}$

Ex: Simplify.

$$\boxed{5.1.7}$$

$$\frac{1}{\sin\alpha - 1} - \frac{1}{\sin\alpha + 1}$$

α = alpha

$$= \frac{1}{\sin\alpha - 1}\left(\frac{\sin\alpha+1}{\sin\alpha+1}\right) - \frac{1}{\sin\alpha+1}\left(\frac{\sin\alpha - 1}{\sin\alpha - 1}\right)$$

Recall:

$A^2 - B^2 = (A+B)(A-B)$

so $\underbrace{(\sin\alpha+1)}_{A+B}\underbrace{(\sin\alpha-1)}_{A-B}$

$= \underbrace{(\sin\alpha)^2}_{A^2} - \underbrace{(1)^2}_{B^2}$

$$= \frac{\sin\alpha + 1}{\sin^2\alpha - 1} - \frac{\sin\alpha - 1}{\sin^2\alpha - 1}$$

$$= \frac{\sin\alpha + 1 - (\sin\alpha - 1)}{\sin^2\alpha - 1}$$

$$= \frac{\sin\alpha + 1 - \sin\alpha + 1}{\sin^2\alpha - 1}$$

Note:

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

$$= \frac{2}{\sin^2\alpha - 1}$$

Also: $A - B = -(B - A)$

$$= \frac{2}{-1(1 - \sin^2\alpha)}$$

$$= \frac{2}{-1(\cos^2\alpha)}$$

$$= -\frac{2}{\cos^2\alpha} = -2\cdot\left(\frac{1}{\cos^2\alpha}\right) = \boxed{-2\sec^2\alpha}$$