5.2: Verifying Identities

5.2.1

We will prove (verify) that an equation is an identity.

(Identity: True for all values of the variable)

To prove $\begin{pmatrix}\text{left}\\ \text{side}\end{pmatrix} = \begin{pmatrix}\text{right}\\ \text{side}\end{pmatrix}$,

our proof should look like:

$$\begin{aligned}
\begin{pmatrix}\text{one}\\ \text{side}\end{pmatrix} &= \\
&= \\
&= \\
&= \\
&= \\
&= \begin{pmatrix}\text{other}\\ \text{side}\end{pmatrix}.
\end{aligned}$$

Tips for proving identities: 5.2.2

* Start with the most complicated side.
* Change to sines and cosines
* If you have 2 or more fractions, find a common denominator and combine them.
* If you have 1 fraction with one term in the denominator, split into separate fractions.
* Use the difference of 2 squares pattern with the Pythagorean identities.
* If you feel stuck:
  * Do SOMETHING!
  * Try the other side

Example: Prove that the equation is an identity.

$$\frac{1-\sin^2\theta}{\cos\theta} = \cos\theta$$

Proof:

$$\frac{1-\sin^2\theta}{\cos\theta} = \frac{\cos^2\theta}{\cos\theta}$$

$$= \frac{\cos\theta \cdot \cancel{\cos\theta}^{\,1}}{\cancel{\cos\theta}_{\,1}}$$

$$= \frac{\cos\theta}{1}$$

$$= \cos\theta \quad \blacksquare$$

Note:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1-\cos^2\theta$$

$$\cos^2\theta = 1-\sin^2\theta$$

Ex: Prove $\cos^2\beta(\tan^2\beta+1)=1$

5.2.3

Proof: $\cos^2\beta\,(\tan^2\beta+1)$

β: Beta

$$= \cos^2\beta\tan^2\beta + \cos^2\beta$$

$$= \cancel{\cos^2}\beta\left(\frac{\sin^2\beta}{\cancel{\cos^2}\beta}\right) + \cos^2\beta$$

$$= \sin^2\beta + \cos^2\beta$$

$$= 1 \quad \blacksquare$$

[from Pythagorean identity]

Ex: Prove.

$$\frac{1-\sin\theta}{1+\sin\theta} = \sec^2\theta - 2\sec\theta\tan\theta + \tan^2\theta$$

5.2.4

Proof: $\frac{1-\sin\theta}{1+\sin\theta}$

$$= \frac{1-\sin\theta}{1+\sin\theta}\left(\frac{1-\sin\theta}{1-\sin\theta}\right)$$

$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$$

$$= \frac{(1-\sin\theta)^2}{\cos^2\theta}$$

$$= \frac{(1-\sin\theta)(1-\sin\theta)}{\cos^2\theta}$$

$$= \frac{1-\sin\theta-\sin\theta+\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1-2\sin\theta+\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1}{\cos^2\theta} - \frac{2\sin\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \sec^2\theta - 2\left(\frac{1}{\cos\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right) + \tan^2\theta$$

$$= \sec^2\theta - 2\sec\theta\tan\theta + \tan^2\theta$$ ~~tat~~

Tip: "multiply by the conjugate"

$A^2 - B^2 = (A+B)(A-B)$

Note: $1-\sin^2\theta$ and $1-\cos^2\theta$ are both a "difference of 2 squares"

To rationalize $\frac{1}{5-\sqrt{2}}$

$\frac{1}{5-\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{5\sqrt{2}-2}$ doesn't work. Instead:

$$\frac{1}{5-\sqrt{2}}\left(\frac{5+\sqrt{2}}{5+\sqrt{2}}\right)$$

$$= \frac{5+\sqrt{2}}{(5)^2-(\sqrt{2})^2}$$

$$= \frac{5+\sqrt{2}}{25-2} = \frac{5+\sqrt{2}}{23}$$