5.5: Double-Angle Identities

$\cos 2A = \cos^2 A - \sin^2 A$
$\sin 2A = 2\sin A \cos A$ } ☆ Know these 2!

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

Verify:

$\sin(2A) = 2\sin A \cos A$

Use
$\sin(A+B) = \sin A\cos B + \cos A \sin B$
with $A=A$, $B=A$

Proof:

$\sin 2A = \sin(A+A)$

$= \sin A\cos A + \cos A \sin A$

$= 2\sin A \cos A$ ▨

Alternative Forms for cos 2A identity

$\cos 2A = \cos^2 A - \sin^2 A$

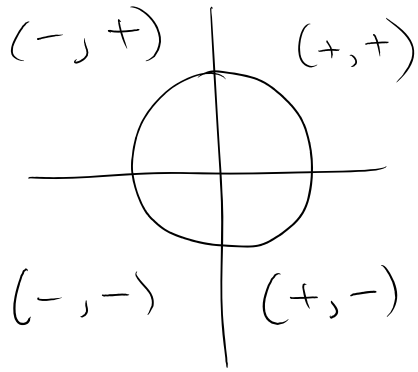
$\cos 2A = 1 - 2\sin^2 A$

$\cos 2A = 2\cos^2 A - 1$

Use
$\cos^2\theta + \sin^2\theta = 1$
$\cos^2\theta = 1 - \sin^2\theta$
$\sin^2\theta = 1 - \cos^2\theta$

Example: Suppose $\cos\theta = -\frac{4}{7}$ and $\sin\theta > 0$. 5.5.2

Find $\cos 2\theta$ and $\sin 2\theta$.



θ has to be in QII

($\cos\theta$ is $-$, $\sin\theta$ is $+$)

$$\cos^2\theta + \sin^2\theta = 1$$

$$\left(-\frac{4}{7}\right)^2 + \sin^2\theta = 1$$

$$\frac{16}{49} + \sin^2\theta = 1$$

$$\sin^2\theta = 1 - \frac{16}{49} = \frac{49}{49} - \frac{16}{49}$$

$$\sin^2\theta = \frac{33}{49}$$

$$\sin\theta = \pm\sqrt{\frac{33}{49}} = \pm\frac{\sqrt{33}}{7}$$

QII, so choose positive

$$\sin\theta = +\frac{\sqrt{33}}{7}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \left(-\frac{4}{7}\right)^2 - \left(\frac{\sqrt{33}}{7}\right)^2 = \frac{16}{49} - \frac{33}{49} = -\frac{17}{49}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\left(\frac{\sqrt{33}}{7}\right)\left(-\frac{4}{7}\right) = -\frac{8\sqrt{33}}{49}$$

$$\boxed{\begin{aligned}\sin 2\theta &= -\frac{8\sqrt{33}}{49}\\ \cos 2\theta &= -\frac{17}{49}\end{aligned}}$$

Previous example cont'd: 5.5.3

Does it make sense for these to be negative?

we know $\theta \in QII$, so

$$90^\circ < \theta < 180^\circ$$
$$2(90^\circ) < 2\theta < 2(180^\circ)$$
$$180^\circ < 2\theta < 360^\circ$$

so quadrant III or IV

Example: Suppose $\cos 2\theta = -\frac{28}{53}$ and $90^\circ < \theta < 180^\circ$. Find $\cos\theta$ and $\sin\theta$

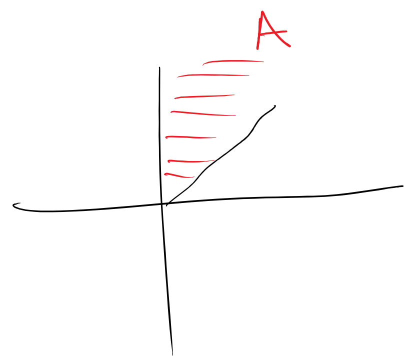
Use $\cos 2A = 2\cos^2 A - 1$

$$-\frac{28}{53} = 2\cos^2 A - 1$$
$$+1 \qquad\qquad +1$$
$$1 - \frac{28}{53} = 2\cos^2 A$$
$$\frac{53}{53} - \frac{28}{53} = 2\cos^2 A$$
$$2\cos^2 A = \frac{25}{53}$$
$$\cos^2 A = \frac{1}{2} \cdot \frac{25}{53} = \frac{25}{106}$$
$$\cos A = \pm\sqrt{\frac{25}{106}} = \pm\frac{5}{\sqrt{106}}$$

Previous example continued:

5.5.4

$2A$ is in Quadrant II



$$90^\circ < 2A < 180^\circ$$

$$\frac{90^\circ}{2} < \frac{2A}{2} < \frac{180^\circ}{2}$$

$$45^\circ < A < 90^\circ$$

so $A \in QI$, so $\cos A$ is positive

$$\cos A = \frac{5}{\sqrt{106}}$$

$$\cos^2 A + \sin^2 A = 1$$

$$\left(\frac{5}{\sqrt{106}}\right)^2 + \sin^2 A = 1$$

$$\sin^2 A = 1 - \frac{25}{106} = \frac{106}{106} - \frac{25}{106}$$

$$\sin^2 A = \frac{81}{106}$$

$$\sin A = \pm\sqrt{\frac{81}{106}} = \pm\frac{9}{\sqrt{106}}$$

QI, so choose positive.

$$\boxed{\sin A = \frac{9}{\sqrt{106}} \quad \cos A = \frac{5}{\sqrt{106}}}$$