Half - Angle Identities

$\boxed{5.6.1}$

$$\left.\begin{aligned} \cos\left(\frac{A}{2}\right) &= \pm\sqrt{\frac{1+\cos A}{2}} \\ \sin\left(\frac{A}{2}\right) &= \pm\sqrt{\frac{1-\cos A}{2}} \end{aligned}\right\}$$

most important ones! (no need to memorize)

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1+\cos A}$$

$$\tan\left(\frac{A}{2}\right) = \frac{1-\cos A}{\sin A}$$

In Calculus II, you will frequently use:

$$\cos^2 A = \frac{1}{2}(\cos 2A + 1)$$

$$\sin^2 A = \frac{1}{2}(\cos 2A - 1)$$

Ex: Determine $\cos\left(\frac{7\pi}{8}\right)$

$$\cos\left(\frac{7\pi}{8}\right) = \cos\left(\frac{7\pi/4}{2}\right)$$

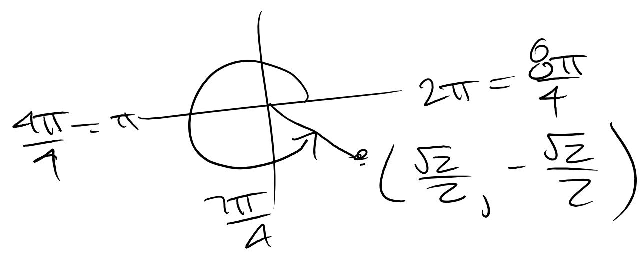
Use identity for $\cos\left(\frac{A}{2}\right)$

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\cos\left(\frac{7\pi/4}{2}\right) = \pm\sqrt{\frac{1+\cos\left(\frac{7\pi}{4}\right)}{2}}$$

[put in $A = \frac{7\pi}{4}$]

Note: $\frac{7\pi/4}{2} = \frac{7\pi}{4}\cdot\frac{1}{2} = \frac{7\pi}{8}$



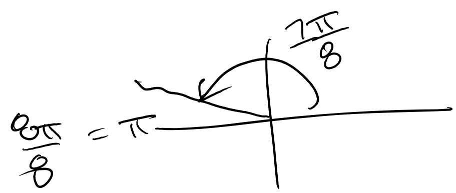
$$= \pm\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$$

$$= \pm\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}\cdot\frac{2}{2}}$$

$$= \pm\sqrt{\frac{2+\sqrt{2}}{4}}$$

$$= \pm\frac{\sqrt{2+\sqrt{2}}}{2}$$

where is $\frac{7\pi}{8}$?



$\frac{7\pi}{8}$ is in QII, so choose the negative

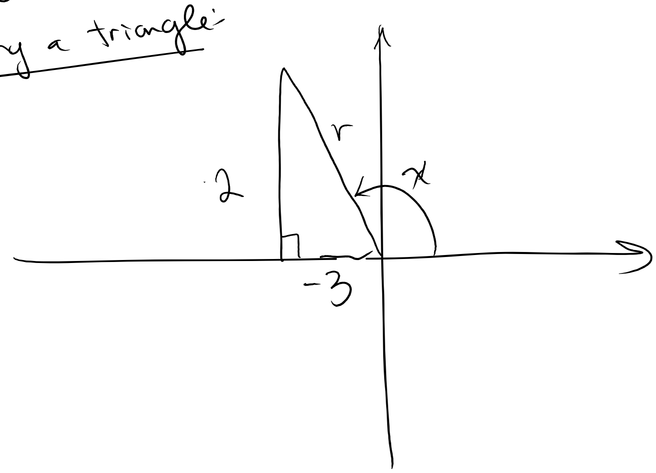
$$\boxed{\cos\left(\frac{7\pi}{8}\right) = -\frac{\sqrt{2+\sqrt{2}}}{2}}$$

Example: Suppose that $\tan x = -\frac{2}{3}$ and that x is in Quadrant II. Find $\cos\left(\frac{x}{2}\right)$.

We want to eventually use the half-angle identity $\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$

To use this identity, we need to find $\cos x$.

using a triangle:



$$r^2 = (2)^2 + 3^2$$
$$= 4 + 9 = 13$$
$$r = \pm\sqrt{13}$$

always choose positive for r.

$$r = \sqrt{13}$$

$$\cos(x) = \frac{-3}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}$$

OR

using Pythagorean Identity

$$\sin^2\theta + \cos^2\theta = 1$$

Divide by $\cos^2\theta$:

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$
$$\tan^2 x + 1 = \sec^2 x$$
$$\left(-\frac{2}{3}\right)^2 + 1 = \sec^2 x$$
$$\frac{4}{9} + 1 = \sec^2 x$$
$$\sec^2 x = \frac{4}{9} + \frac{9}{9} = \frac{13}{9}$$
$$\sec x = \pm\sqrt{\frac{13}{9}} = \pm\frac{\sqrt{13}}{3}$$
$$\cos x = \pm\frac{3}{\sqrt{13}}$$

Quadrant II, so choose

$$\cos x = -\frac{3}{\sqrt{13}}$$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$$
$$= \pm\sqrt{\frac{1-\frac{3\sqrt{13}}{13}}{2}} = \pm\sqrt{\frac{1-\frac{3\sqrt{13}}{13}}{2}\cdot\frac{13}{13}}$$

See next page

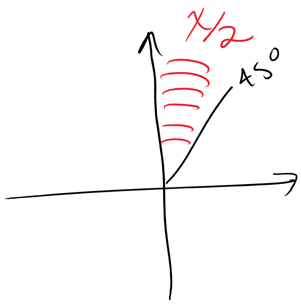
5.6.4

$$= \pm\sqrt{\frac{13-3\sqrt{13}}{26}}$$

still need to decide + or −

x is in QII, so

$$90^\circ < x < 180^\circ$$



$$\frac{90^\circ}{2} < \frac{x}{2} < \frac{180^\circ}{2}$$

$$45^\circ < \frac{x}{2} < 90^\circ$$

So, $\frac{x}{2}$ is in QI. Therefore $\cos\left(\frac{x}{2}\right)$ is positive

$$\boxed{\cos\left(\frac{x}{2}\right) = \sqrt{\frac{13-3\sqrt{13}}{26}}}$$