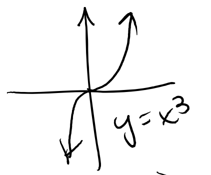
6.1: The Inverse Trigonometric Functions (Inverse Circular Functions)

Recall:

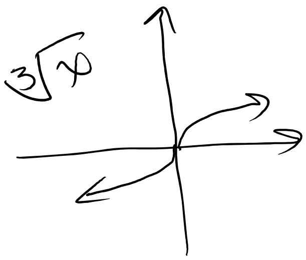
What is the inverse function of $f(x) = x^3$?

x	$y = x^3$
-3	-27
-2	-8
-1	-1
0	0
1	1
2	8
3	27



x	$f^{-1}(x) = \sqrt[3]{x}$
-27	-3
-8	-2
-1	-1
0	0
8	2
27	3

$y = \sqrt[3]{x}$



A function is one-to-one if every input in the domain is associated with exactly (output) in the range.

A function is one-to-one if the graph passes the horizontal line test (no horizontal line intersects it more than once).

If a function is one-to-one, it has an inverse function

6.1.2

Does $f(x) = x^2$ have an inverse?

x	$y = x^2$
-2	$(-2)^2 = 4$
-1	$(-1)^2 = 1$
0	$0^2 = 0$
1	$1^2 = 1$
2	$2^2 = 4$

$\Longrightarrow$

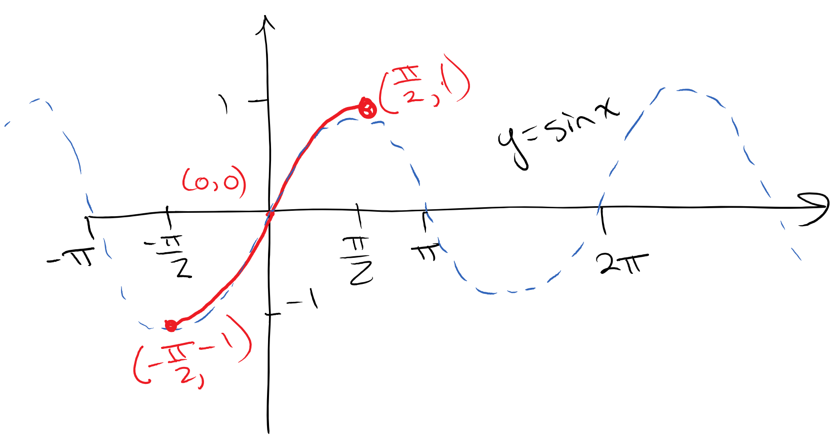
x	
4	-2
1	-1
0	0
1	1
4	2

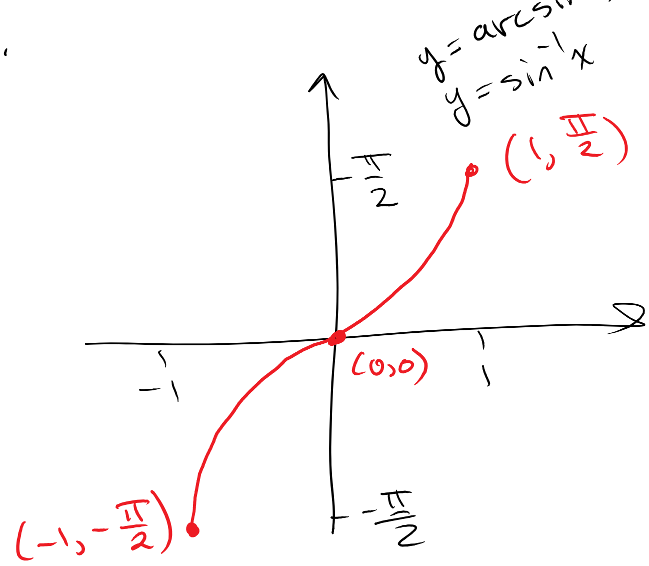
not a function

Important: No trig function (on its usual domain) is one-to-one. Trig functions don't pass the horizontal line test.

Inverse Sine Function

(It's the inverse of a modified-domain version of the sine function).



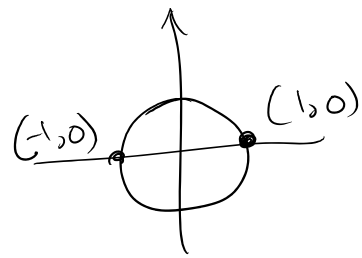


Definition:

$y = \sin^{-1} x$ if and only if $x = \sin y$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

(equivalently, $y = \arcsin x$)

6.1.3



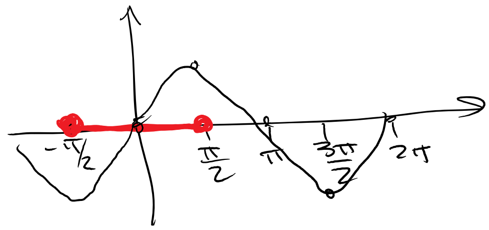
Ex: Find $\sin^{-1}(0)$.

Let $\theta = \sin^{-1}(0)$.

Then $\sin\theta = 0$ and $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\sin\theta = 0$ for $\theta = 0, \theta = \pi, \theta = 2\pi, \dots$

Only 0 is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



$$\boxed{\sin^{-1}(0) = 0}$$

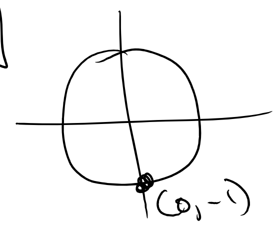
Ex: Determine $\sin^{-1}(-1)$.

Let $\theta = \sin^{-1}(-1)$.

Then $\sin\theta = -1$ and $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin\left(\frac{3\pi}{2}\right) = -1$, but $\frac{3\pi}{2} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

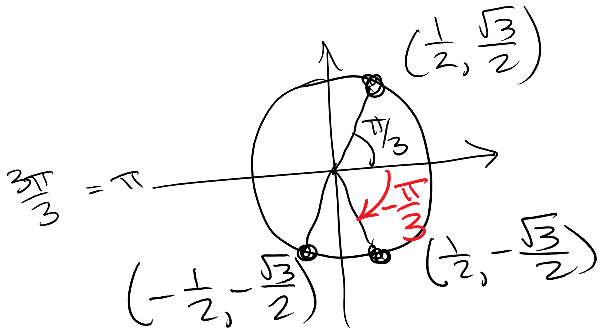
$$\boxed{\sin^{-1}(-1) = -\frac{\pi}{2}}$$



Ex: Find $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$.

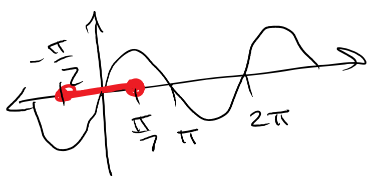
Let $\theta = \arcsin\left(-\frac{\sqrt{3}}{2}\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Then $\sin\theta = -\frac{\sqrt{3}}{2}$ and $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



angles that have $\sin\theta = -\frac{\sqrt{3}}{2}$ include

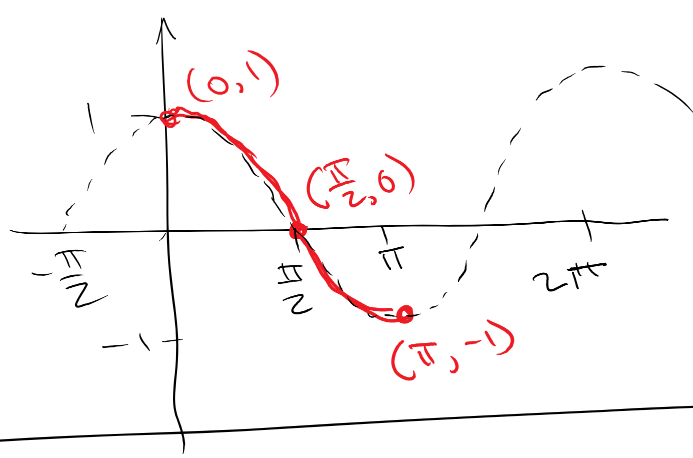
$\frac{4\pi}{3}, \frac{5\pi}{3}, -\frac{\pi}{3}, -\frac{2\pi}{3}$

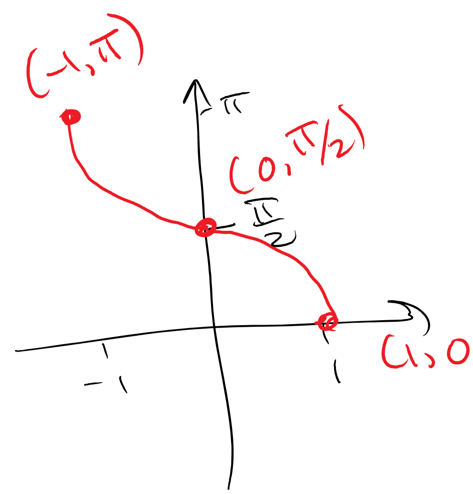


only $-\frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

so $\boxed{\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}}$

Inverse Cosine Function

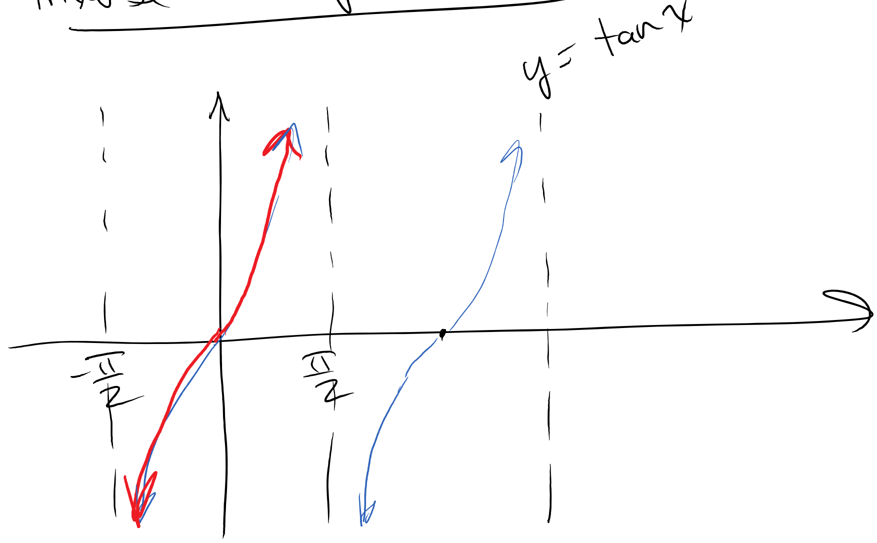


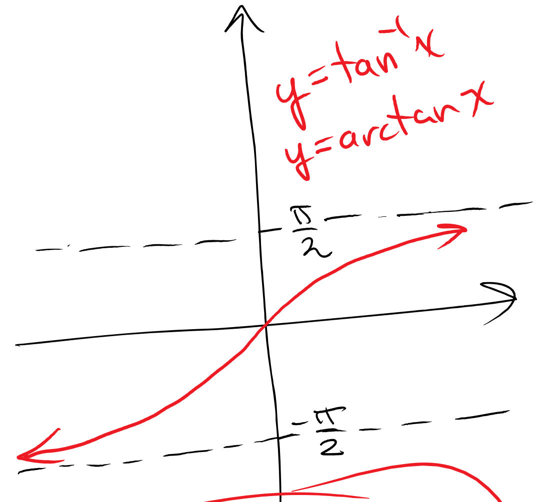


Def'n: $y = \cos^{-1} x$ (or $\arccos x$) if and only if $x = \cos y$ and $0 \le y \le \pi$.

6.1.5

Inverse Tangent Function





Know this graph!

Def'n: $y = \tan^{-1} x$ (or $\arctan x$) if and only if $x = \tan y$ and $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$