7.1.1

7.1: The Law of Sines

The law of Sines (7.1 and 7.2) and the Law of Cosines (7.3) let us solve non-right (oblique) triangles.

Recall: To solve a triangle means to find the missing sides and angles.

Given Info	Use
SAA or ASA	Law of Sines
SSA	Law of Sines - "ambiguous case" there can be 1 triangle, 2 triangles, or no triangle.
SAS	Law of Cosines
SSS	Law of Cosines

7.1.2

Law of Sines

The lengths of the sides are proportional to the sines of the angles opposite them.
(for any triangle, not just right triangles)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

(a opposite A, b opposite B, c opp. C)

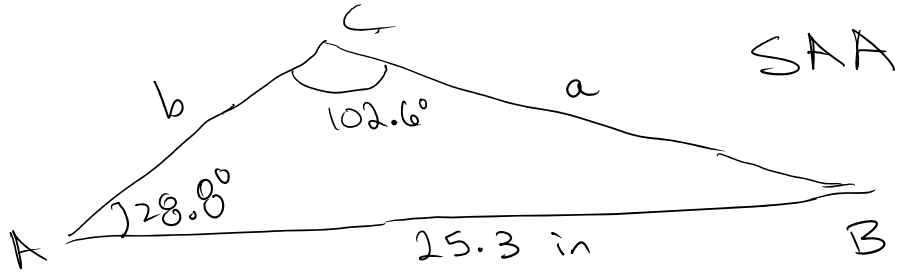
or equivalently,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example: Solve triangle ABC if 7.1.3

$A = 28.8^\circ$, $C = 102.6^\circ$, and $c = 25.3$ in

SAA or AAS



$B = 180^\circ - 102.6^\circ - 28.8^\circ = 48.6^\circ$

ASA

~~SAS~~

Note: This turns it into ~~SAS~~ (so SAS and SAA are almost equivalent, because we can get the 3rd angle so easily)

(For SAA or ASA, use Law of Sines)

$$\frac{\sin 102.6^\circ}{25.3 \text{ in}} = \frac{\sin 28.8^\circ}{a}$$

$$\frac{25.3 \text{ in}}{\sin 102.6^\circ} = \frac{a}{\sin 28.8^\circ}$$

$$\frac{(\sin 28.8^\circ)\, 25.3 \text{ in}}{\sin 102.6^\circ} = \frac{a}{\sin 28.8^\circ}\,(\sin 28.8^\circ)$$

$$a = \frac{\sin 28.8^\circ\,(25.3 \text{ in})}{\sin 102.6^\circ} \approx 12.489 \text{ in}$$

see next page

Now find b: $\frac{\sin 48.6^\circ}{b} = \frac{\sin 102.6^\circ}{25.3}$ 7.1.4

$$\frac{b}{\sin 48.6^\circ} = \frac{25.3 \text{ in}}{\sin 102.6^\circ}$$

$$b = \frac{25.3 \text{ in}}{\sin 102.6^\circ} \cdot \sin 48.6^\circ$$

$$b \approx 19.446$$

$B = 48.6^\circ$

$b = 19.4$ in

$a = 12.5$ in

7.1.5

Area of a Triangle

For any triangle ABC, the Area is

$$\text{Area} = \frac{1}{2}bc\sin A$$

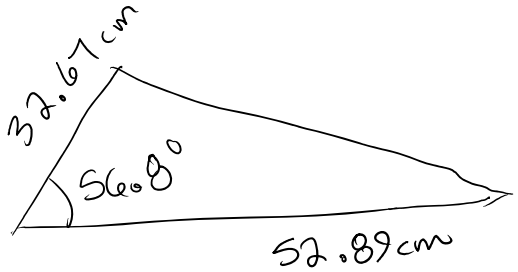
Equivalently, $A = \frac{1}{2}ab\sin C$

$A = \frac{1}{2}ac\sin B$

(A opposite a, etc)

(SAS area formula)

Ex: Find the area of triangle ABC, if $A = 56.8°$, $b = 32.67$ cm, $c = 52.89$ cm



$$\text{Area} = \frac{1}{2}(32.67\text{cm})(52.89\text{cm})\sin 56.8°$$

$$\text{Area} = 722.93 \text{ cm}^2$$