Tuesday, April 23, 2019 9:38 AM

7.2: Ambiguous Case of the Law of Sines

7.2.1

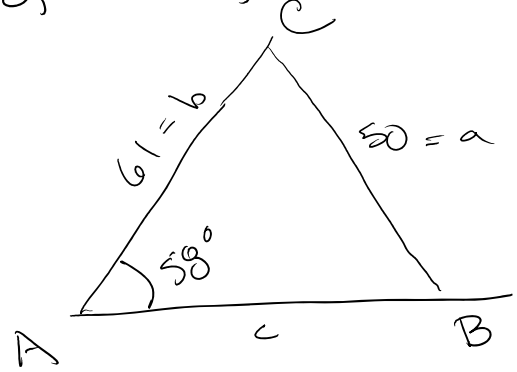
Summary – Solving Triangles

Given Info	Tool
SAA or ASA	Law of Sines
SSA	Law of Sines "ambiguous case" There may be two triangles, one triangle, or no triangle at all that fit the given info
SAS	Law of Cosines
SSS	Law of Cosines

7.2.2

Example:

Solve the triangle, given that $a = 50$, $b = 61$, $A = 58°$.



SSA
(use Law of Sines)

$$\frac{\sin 58°}{50} = \frac{\sin B}{61}$$

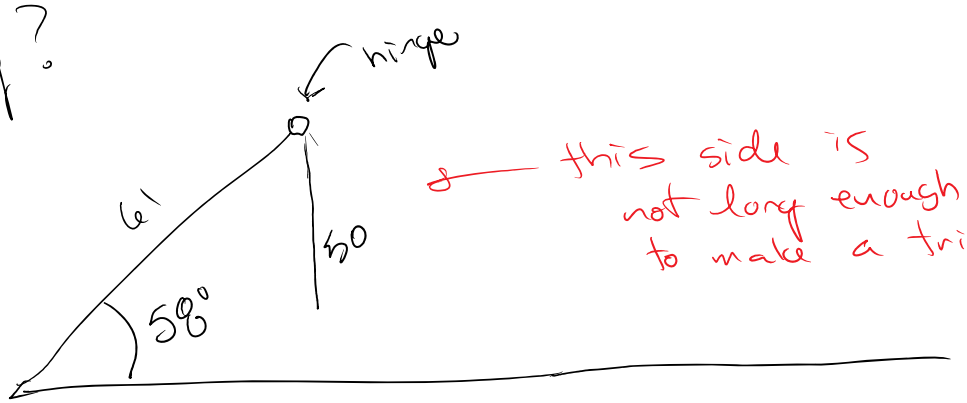
$$61\left(\frac{\sin 58°}{50}\right) = \sin B$$

$$\sin B \approx 1.0346$$

Impossible! $\quad -1 \leq \sin B \leq 1$

There is no triangle that satisfies this information

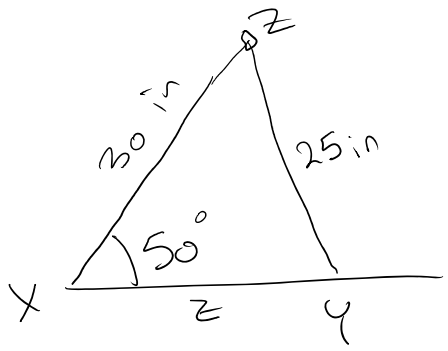
Why?



← this side is not long enough to make a triangle

7.2.3

Example: Solve the triangle XYZ given that $X = 50°$, $x = 25$ in, $y = 30$ in.

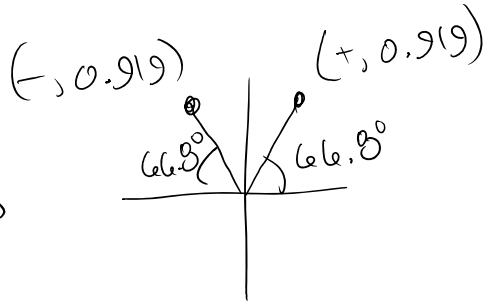


SSA

$$\frac{\sin 50°}{25 \text{ in}} = \frac{\sin Y}{30 \text{ in}}$$

$$30 \text{ in}\left(\frac{\sin 50°}{25 \text{ in}}\right) = \sin Y$$

$$\sin Y \approx 0.919253$$



Ref. angle is $Y = \sin^{-1}(0.919253)$

$\approx 66.8°$

There are 2 possible angles that solve $\sin Y = 0.919253$

Q1 angle: Ref angle $Y = 66.8°$

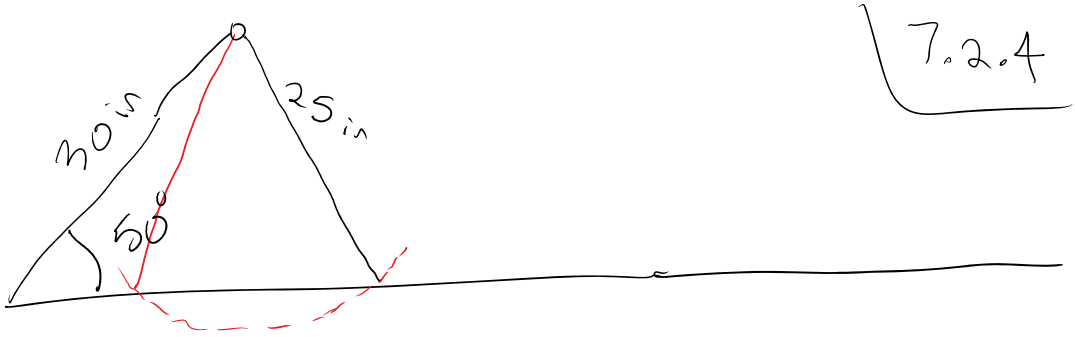
Q2 angle: $180° - 66.8° = 113.2°$ (supplement of $66.8°$)

Can we make a triangle using $Y = 113.2°$?

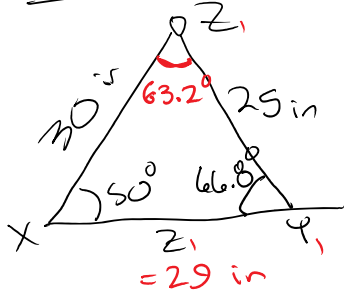
$113.2° + 50° = 163.2° < 180°$, so yes, it can work for a triangle.

(If the supplement + given angle $> 180°$, then the supplement can't work for our triangle)

7.2.4



1st triangle:



$Z = 180° - 50° - 66.8° = 63.2°$

$$\frac{\sin 63.2°}{z} = \frac{\sin 50°}{25\text{ in}}$$

$$\frac{z}{\sin 63.2°} = \frac{25\text{ in}}{\sin 50}$$

$$z = \frac{25\text{ in}}{\sin 50}(\sin 63.2°) = 29.13\text{ in}$$

Triangle #1

$Y_1 = 66.8°$

$Z_1 = 63.2°$

$z_1 = 29$ in

From earlier,

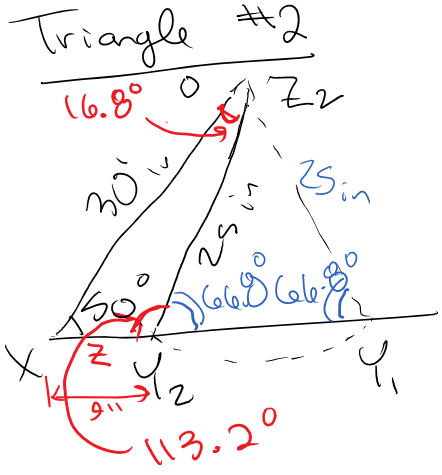
$Y_2 = 180° - 66.8° = 113.2°$

$Z_2 = 180° - 113.2° - 50° = 16.8°$

$$\frac{z}{\sin 16.8°} = \frac{25\text{ in}}{\sin 50°}$$

$$z = \frac{25\text{ in}}{\sin 50}(\sin 16.8°) \approx 9.13\text{ in}$$

Triangle #2



Triangle #2: $Z_2 = 16.8°$, $Y_2 = 113.2°$, $z = 9$ in