7.3 : Law of Cosines

Law of Cosines

In any triangle ABC, it is true that:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

(A across from a, etc)

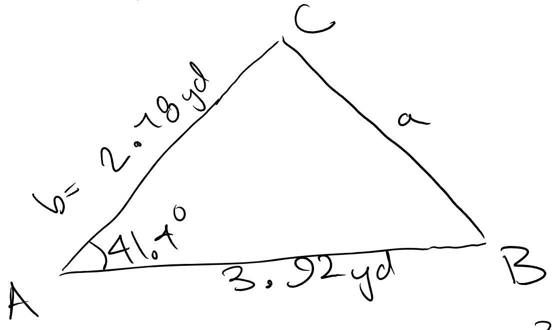
Equivalently, $a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

Use Law of Cosines for SAS and SSS.

Example: Solve the triangle, given that $\boxed{7.3.2}$

$A = 41.4°$, $b = 2.78$ yds, $c = 3.92$ yds.



SAS
↳ Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = (2.78yd)^2 + (3.92yd)^2 - 2(2.78yd)(3.92yd)\cos 41.4°$$

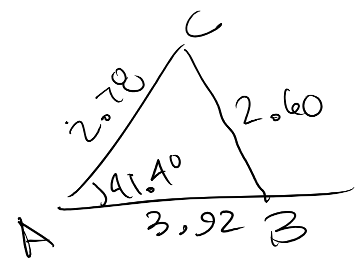
$$a^2 = 6.74560$$

$$a = \pm\sqrt{6.74560} \approx \pm 2.5973$$

side length, so must be positive

$$a = 2.5973$$

$$a \approx 2.60$$



If you use law of sines for the remaining angle, find the smaller angle 1st
(smaller angle is across from smaller side)
(smaller angle cannot be obtuse)

Or, use Law of Cosines.

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7.3.3

Or use Law of Cosines:

$$(3.92)^2 = (2.60)^2 + (2.78)^2 - 2(2.78)(2.6) \cos C$$

$$15.3664 = 14.4884 - 14.456 \cos C$$

$$0.878 = -14.456 \cos C$$

$$-0.0607 = \cos C$$

$$C = \cos^{-1}(-0.0607) \approx 93.5^\circ$$

Then find 3rd angle: $180^\circ - 93.5^\circ - 41.4^\circ = 45.1^\circ$

$$\boxed{\begin{array}{l} C = 93.5^\circ \\ B = 45.1^\circ \\ a = 2.60 \text{ yd} \end{array}}$$

Area:

2 area formulas:

SAS area $\quad \text{Area} = \frac{1}{2}ab\sin C$

SSS Area (Heron's formula)

semiperimeter: $s = \frac{1}{2}(a+b+c)$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$