8.2: Union, Intersecion, and Complement of Events; Odds

Unions and Intersections:

Example 1: Roll a single die.

a. What is the probability of rolling a number that is even and divisible by 3?
b. What is the probability of rolling a number that is even or divisible by 3?

(a) Sample space: $S = \{1,2,3,4,5,6\}$

$A = \{6\}$

$P(A) = \frac{n(A)}{n(S)} = \boxed{\frac{1}{6}}$

(b) $B = \{2,3,4,6\}$

$P(B) = \frac{n(B)}{n(S)} = \frac{4}{6} = \boxed{\frac{2}{3}}$

Another approach:

Let E = set of Even Numbers = $\{2,4,6\}$

T = set of numbers divisible by 3 = $\{3,6\}$

(a) and

$E \cap T = \{6\}$

$P(E \cap T) = \boxed{\frac{1}{6}}$

(b) or

$P(E \cup T) = P(E) + P(T) - P(E \cap T)$

$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6}$

$= \frac{4}{6} = \boxed{\frac{2}{3}}$

Recall: Addition Rule for counting:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

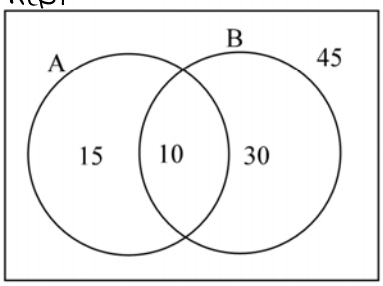
Probability of a Union of Two Events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If the two events are mutually exclusive (disjoint):

$$P(A \cup B) = P(A) + P(B)$$

Example 2: Assume that an equally likely sample space is described by the Venn diagram below.



Find these probabilities:

$P(A) = \frac{15+10}{100} = \frac{25}{100} = \boxed{0.25}$

$P(B) = \frac{10+30}{100} = \frac{40}{100} = \boxed{0.40}$

$P(A \cap B) = \frac{10}{100} = \boxed{0.10}$

$P(A \cup B) = \frac{15+10+30}{100} = \frac{55}{100} = \boxed{0.55}$

Complements:

Probability of a complement:
$P(E') = 1 - P(E)$
$P(E) = 1 - P(E')$

Example 3: Suppose that the probability of someone voting for a certain candidate is 0.46. What is the probability of not voting for the candidate?

$P(E) = 0.46$

$P(E') = 1 - 0.46 = \boxed{0.54}$ = prob of not voting for the candidate.

Example 4: Roll a pair of dice. What is the probability of rolling a sum of 4 or more?

Sample Space:

$S = \{(1,1)\ (1,2)\ (1,3) \ldots (1,6),\ (2,1)\ (2,2)\ (2,3) \ldots (2,6),\ \vdots,\ (6,1) \ldots (6,6)\}$

$n(S) = 6 \cdot 6 = 36$

E: we roll a sum of 4 or more

E': sum is less than 4

$E' = \{(1,1), (1,2), (2,1)\}$

$P(E') = \frac{n(E')}{n(S)} = \frac{3}{36}$

$P(E) = 1 - \frac{3}{36}$

$= \frac{36}{36} - \frac{3}{36} = \frac{33}{36}$

$= \boxed{\frac{11}{12}}$

Odds:

Sometimes the likelihood (or unlikelihood) of an event is described using *odds* instead of probabilities.

Summary:
Probability: The event is contrasted against the whole.
Odds: The event is contrasted against the complement.

Ex 4 $\frac{1}{2}$: What are the odds for rolling a sum of 4 or more when rolling two dice?

Odds for E are $\frac{33}{3}$ or $\boxed{\frac{11}{1}}$, usually written $\boxed{11:1}$ (write it as a ratio)

Odds against rolling a sum of 4 or more are 3:33 or $\frac{3}{33}$ or $\boxed{\frac{1}{11} \text{ or } 1:11}$

Converting from probability to odds:

From Probability to Odds:

- Odds for $E = \dfrac{P(E)}{P(E')}$
- Odds against $E = \dfrac{P(E')}{P(E)}$

When possible, express odds as ratios of whole numbers.

or use formula:

$P(E) = \frac{2}{36}$

$P(E') = 1 - \frac{2}{36} = \frac{34}{36}$

Odds for E are

$\frac{P(E)}{P(E')} = \frac{2/36}{34/36} = \frac{2}{36} \cdot \frac{36}{34} = \frac{2}{34} = \frac{1}{17}$

Example 5: Roll a pair of dice. What are the odds for rolling a sum of 3? What are the odds against rolling a sum of 3?

E: Sum is 3

$E = \{(1,2), (2,1)\}$, $n(S) = 36$

$n(E) = 2$

$n(E') = 34$

Odds for E are $\frac{2}{34}$, so $\frac{1}{17}$ or 1:17.

odds against E are $\frac{34}{2}$, so $\frac{17}{1}$ or 17:1.

Example 6: What are the odds against ~~rolling~~ drawing an ace when drawing a single card from a standard deck?

Sample Space: S = set of all 52 cards

$n(S) = 52$

A: an ace is drawn

$n(A) = 4$

$A = \{A♥, A♦, A♣, A♠\}$

so $n(A') = 48$

odds against A are $\frac{48}{4} = \frac{12}{1}$ or 12:1

Example 7: Suppose that, based upon genetics, a child has a 0.08 probability of developing a certain disease. What are the odds against the child developing the disease?

E: Child gets the disease

$P(E) = 0.08$

$P(E') = 0.92$

odds against E are $\frac{0.92}{0.08} = \frac{92}{8} = \frac{46}{4} = \frac{23}{2}$

or 23:2

or 11.5:1

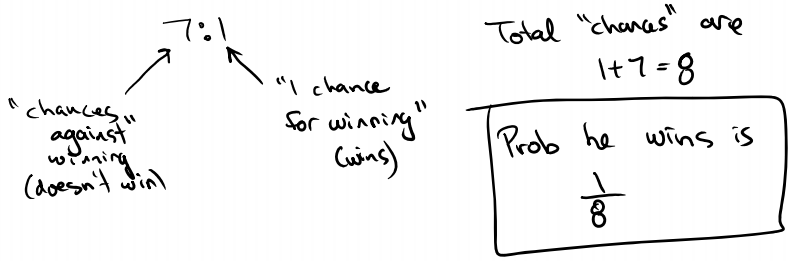
$\frac{23}{2} = 11.5 = \frac{11.5}{1}$

Converting odds to probability:

From Odds to Probability:

If odds for an event E are $\frac{m}{n}$, (i.e. m:n) then $P(E) = \frac{m}{m+n}$.

Example 8: If the odds against a horse winning a race are 7:1, what is the probability that the horse will win?



Example 9: Suppose an insurance company has used past flood data to determine that determined that the odds against a particular house flooding are 150:1. What is the probability that the house floods?