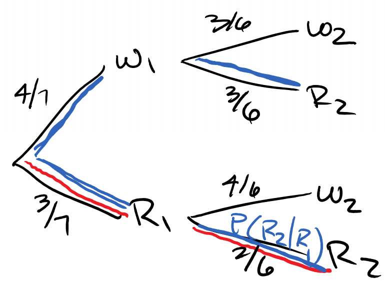
8.4: Bayes' Formula

Previously, when we discussed conditional probabilities, we considered the probability of a "later" event (in a probability tree), given that an "earlier" event occurred.

Now, we will consider the probability of an event "earlier" in the tree, given that a "later" event occurs.

Example 1: A box contains 4 white marbles and 3 red marbles. Two marbles are drawn in succession without replacement. What is the probability of drawing a red marble on the first draw given that a red marble is drawn on the second draw?

WWWW
RRR



We need to find $P(R_1 | R_2)$

Recall:
Conditional Probability Definition:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(R_1|R_2) = \frac{P(R_1 \cap R_2)}{P(R_2)}$$

$$= \frac{\frac{3}{7}\left(\frac{2}{6}\right)}{\frac{3}{7}\left(\frac{2}{6}\right) + \frac{4}{7}\left(\frac{3}{6}\right)}$$

$$= \frac{\frac{6}{42}}{\frac{6}{42} + \frac{12}{42}} = \frac{\frac{6}{42}}{\frac{18}{42}} = \frac{6}{\cancel{42}} \cdot \frac{\cancel{42}}{18} = \frac{6}{18}$$

$$= \boxed{\frac{1}{3}}$$

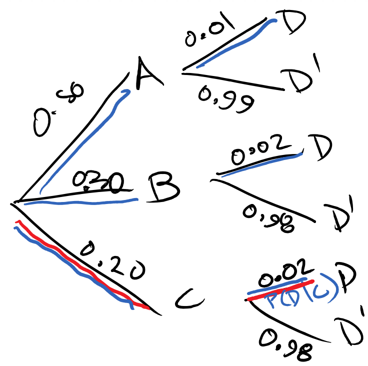
We've just used something a theorem known as *Bayes' Theorem*. You don't need to memorize it, as it is just the conditional probability formula $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$.

<u>Bayes' Formula</u>:

$$P(A|B) = \frac{\text{product of probabilities on path to B through A}}{\text{sum of all branch products leading to B}}$$

Example 2: The picture tubes for the Pulsar 19-inch television sets are manufactured in three locations and then shipped to the main plant for final assembly. Plants A, B, and C supply 50%, 30%, and 20%, respectively, of the picture tubes. The quality-control department of the company has determined that 1% of the picture tubes produced by plant A are defective, whereas 2% of the tubes produced by plants B and C are defective. If a Pulsar 19-inch color television is selected at random and the picture tube is found to be defective, what is the probability that the picture tube was manufactured in plant C?

D: Defective



we want to find

$$P(C|D)$$

$$P(C|D) = \frac{P(C \cap D)}{P(D)}$$

$$= \frac{0.20(0.02)}{0.50(0.01) + 0.30(0.02) + 0.20(0.02)}$$

$$= \frac{0.004}{0.015} \approx \boxed{0.02667}$$

Monday, April 1, 2019 1:04 PM

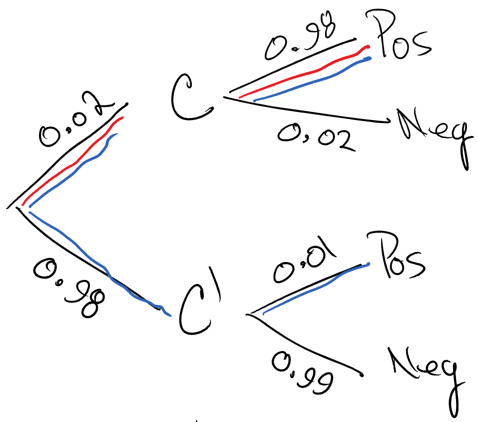
8.4 # 57 Cancer test

C: has cancer

C': doesn't have cancer

Pos: test shows cancer

Neg: test doesn't show cancer



ⓐ Find prob that person has cancer given the test shows cancer.

$$P(C \mid Pos) = \frac{P(C \cap Pos)}{P(Pos)} = \frac{0.02(0.98)}{0.02(0.98) + 0.98(0.01)}$$

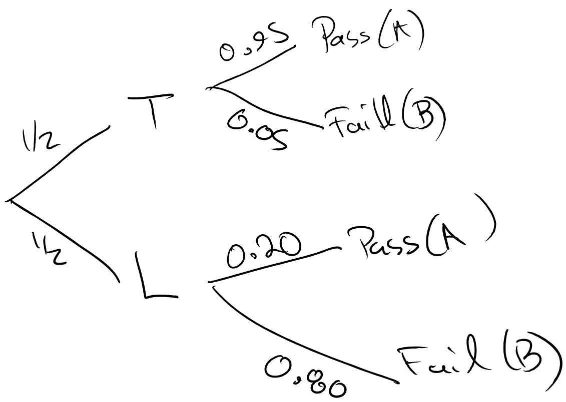
8.4 # 61) Lie detector

T: person telling truth

L: person lying

Pass: A: lie detector test says tell truth

Fail: B: lie detector says they lie



ⓐ $P(L \mid Fail)$