3.4: Present Value of an Annuity; Amortization

Present Value of an Ordinary Annuity
$$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$$
where
$PMT =$ periodic payment (made at end of period)
$i = \frac{r}{m} =$ rate per period
$n =$ number of payments (periods)
$PV =$ present value of all payments

Example 1: How much should you deposit (now) into an account that pays 6% compounded semiannually so that \$1,000 may be withdrawn every 6 months for three years?

$$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$PV = \$1000\left[\frac{1-\left(1+\frac{0.06}{2}\right)^{-6}}{\frac{0.06}{2}}\right]$$

$$= \boxed{\$5417.19}$$

$PV = ?$

$PMT = \$1000$

$i = \frac{r}{m} = \frac{0.06}{2}$

$r = 0.06$

$m = 2$ (semiannual compounding)

$n = mt = 2(3) = 6$

The *amortization* of a debt is the process of paying it off in equal installments. For example, if I buy a new car and don't have the cash for it, I *amortize* the debt by making equal monthly payments.

Example 2: Suppose you want to finance an \$800 television. The electronics store is willing to finance it for 18 months at 18% compounded monthly.

a. What are the monthly payments?
b. How much total interest will you pay?

Example 3: I buy a car for \$20,000. I put \$800 down and the dealer gives me \$1800 for my trade-in. I finance the rest at 5.5% for five years (compounded monthly). What are my monthly payments? How much total money do I pay for the car? How much interest?

Total Amount Financed = \$20 000 − \$800 − \$1800 = \$17 400

Use PV formula.

$$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$$

PV = \$17 400
PMT = ?
$i = \frac{r}{m} = \frac{0.055}{12}$
$n = mt = 12(5) = 60$
$r = 0.055$
$m = 12$
$t = 5$

$$\$17\,400 = PMT\left[\frac{1-\left(1+\frac{0.055}{12}\right)^{-60}}{\frac{0.055}{12}}\right]$$

PMT = \$332.36

$\boxed{\text{Monthly payment is } \$332.36}$

Total Paid: 60(\$332.36) = \$19 941.60

Total amount paid is \$19 941.60 + \$800 + \$1800 = $\boxed{\$22\,541.60}$

How much is interest!
Total of payments − amount financed = \$19 941.60 − \$17 400
= $\boxed{\$2541.60 \text{ interest}}$

Example 4: Scott and Jennifer are considering buying a house. The house they like costs \$110,000, and they have saved \$10,000 for a down payment.

a. What will be their monthly payment for a 30-year loan at 5% (compounded monthly)? How much interest will they pay?

b. What will be their monthly payment for a 15-year loan at 5% (compounded monthly)? How much interest will they pay?

c. What will be their monthly payment for a 15-year loan at 4.6% (compounded monthly)? How much interest will they pay?

a

$$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$$

30-year loan: PMT = \$536.82

Total Interest: \$93 255.20

b 15-year loan: PMT = \$790.79

Total Interest: \$42 342.20

Summary of formulas:

- Single Payment (Lump Sum just sitting)
  - Simple Interest $A = P(1+rt)$
  - Compound Interest $A = P\left(1+\frac{r}{m}\right)^n$
- Sequence of Payments
  - Involves Value <u>after</u> Payments $FV = PMT\left[\frac{(1+i)^n - 1}{i}\right]$
  - Involves Value <u>before</u> Payments $PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$

Amortization schedules:

<u>Example 5:</u> Suppose that Scott and Jennifer decided to buy the $110,000 house with the $10,000 down payment, financed for 30 years at 5%. How much of their first 3 payments went to interest? How much would have gone toward interest had they opted for the 15-year loan at 5%?

$i = \frac{r}{m} = \frac{0.05}{12} \approx 0.00417$

PMT: $536.82 (From Ex 4a)

<u>30-year loan</u>

<u>1st period:</u>

$0.00417(\$100\,000) = \$416.67$

PMT: $\$536.82 = \underbrace{\$416.67}_{\text{interest}} + \underbrace{\$120.15}_{\text{Principal}}$

<u>2nd Period:</u>

Principal $= \$100\,000 - \$120.15$

$= \$99\,879.85$

Interest: $0.00417(\$99\,879.85) = \$416.17$

PMT #2: $\$536.82 = \underbrace{\$416.17}_{\text{Interest}} + \underbrace{\$120.65}_{\text{Principal}}$

Equity after 2 payments: $\$120.15 + \$120.65$

$= 240.80$

Example 6: Scott and Jennifer ended buying a \$135,000 house. They chose the 30-year mortgage at 6% annual interest rate, with a 3% down payment. Six years later, they were able to refinance at 3.875% with no closing costs. This time, they chose a 15-year mortgage.

a. What was their remaining balance when they refinanced?
b. How much interest did they pay during the first six years (on the original mortgage)?
c. How much interest would they have paid during the remaining 24 years of the original mortgage?
d. What are the payments on the new mortgage?
e. Suppose that after paying on the new mortgage for nine years (with no additional principal payments), the house is worth \$200,000. How much equity do they have?

2003: Bought house for \$135K, 30yrs, 6%, 3% down

Down Pmt: $0.03(\$135000) = \$4050$

$PV = \$135000 - \$4050 = \$130\,950$

$$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right] \qquad i = \frac{r}{m} = \frac{0.06}{12}$$

$$n = 12(30) = 360$$

$$130\,950 = PMT\left[\frac{1-\left(1+\frac{0.06}{12}\right)^{-360}}{\frac{0.06}{12}}\right]$$

$$PMT = \$785.11$$

2009: Decide to refinance to a 15-yr at 3.875%

Need to find the remaining balance. This will be the principal for the new loan

6 years have passed, so 24 years remain on the 30-year mortgage.

We need the PV of a 24-year sequence of \$785.11 payments

$$PV = PMT\left[\frac{1-(1-i)^{-n}}{i}\right] \qquad PMT = \$785.11$$

$PV = ?$

$n = 24(12) =$

$$= \$785.11\left[\frac{1-\left(1+\frac{0.06}{12}\right)^{-288}}{i}\right]$$

$$= \$119,685.40$$