5.2: Systems of Linear Inequalities in Two Variables

Solving systems of linear inequalities graphically:

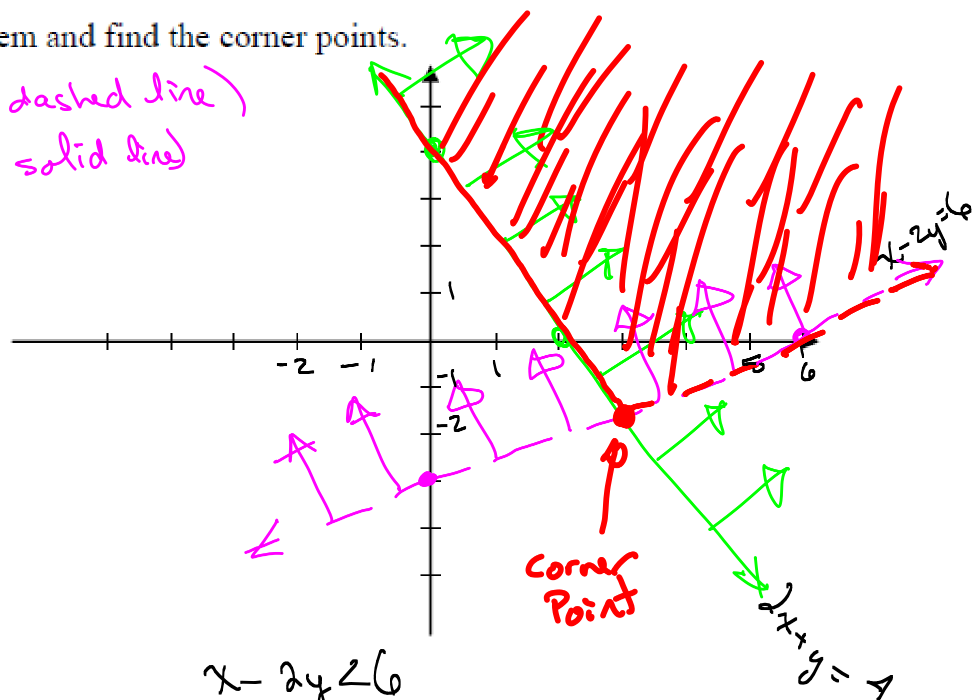
Now we'll solve systems of several inequalities.

We want to find the graph of all order pairs (x, y) that simultaneously satisfy all the inequalities in the system. The graph is called the *solution region*, or the *feasible region*, for the system. To find the solution region, we graph each inequality in the system (this will give a shaded area for each). The area that is included in *all* of them is the feasible region.

A *corner point* of a feasible region is a point in the solution region that is the intersection of two boundary lines.

Example 1: Solve the following system and find the corner points.

$$x - 2y < 6 \quad (< \Rightarrow \text{dashed line})$$
$$2x + y \geq 4 \quad (\geq \Rightarrow \text{solid line})$$



Graph the lines:

$x - 2y = 6 \Rightarrow (0, -3)\ (6, 0)$
$2x + y = 4 \Rightarrow (0, 4)\ (2, 0)$

For $x - 2y = 6$
To find x-intercept, set $y = 0$:
$x - 2(0) = 6$
$x - 0 = 6$
$x = 6 \Rightarrow (6, 0)$

To find y-intercept, set $x = 0$
$0 - 2y = 6$
$-2y = 6$
$y = -3 \Rightarrow (0, -3)$

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$x - 2y < 6$
Test point: $(0, 0)$
$0 - 2(0) < 6$
$0 < 6$ True. Shade half that contains $(0, 0)$

$2x + y \geq 4$
Test point: $(0, 0)$
$2(0) + 0 \geq 4$
$0 \geq 4$ False. Shade half not containing $(0, 0)$

Ex 1 cont'd:

Find the corner point:

$$x - 2y = 6 \xrightarrow{(-2)} -2x + 4y = -12$$
$$2x + y = 4 \longrightarrow 2x + y = 4$$

Add: $5y = -8$

$$y = -\frac{8}{5} = -1\frac{3}{5} = -1.6$$

Plug in $y = -1.6$ or, do elimination again:

$$x - 2y = 6 \longrightarrow x - 2y = 6$$
$$2x + y = 4 \xrightarrow{(2)} 4x + 2y = 8$$

$$5x = 14$$

$$x = \frac{14}{5} = 2\frac{4}{5} = 2.8$$

Corner point: $\left(2\frac{4}{5}, -1\frac{3}{5}\right)$ or $(2.8, -1.6)$

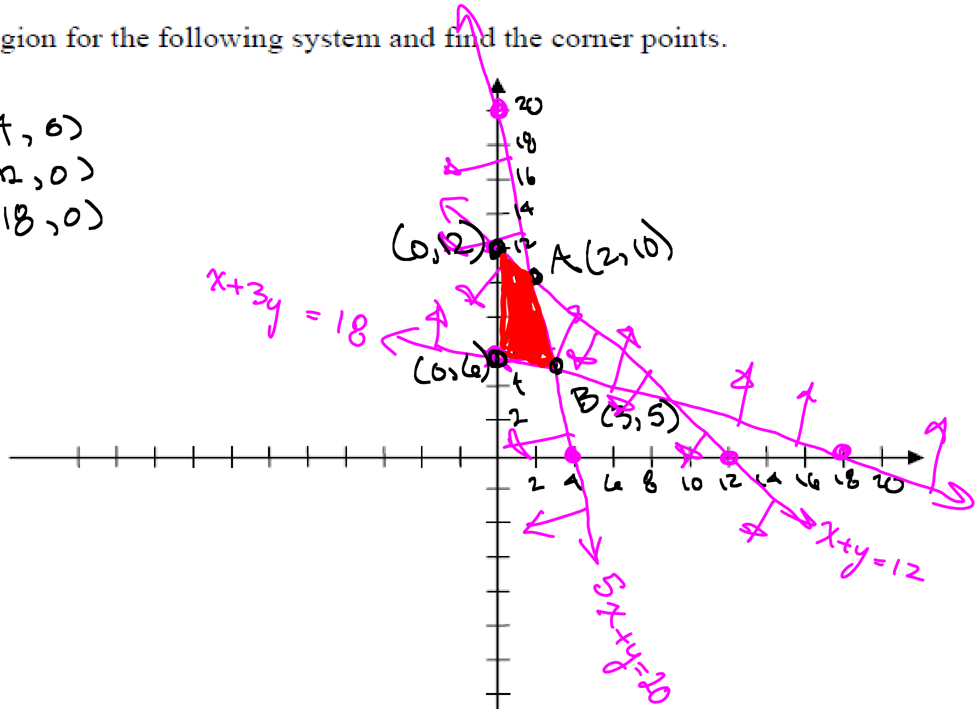
Example 2: Graph the feasible region for the following system and find the corner points.

$5x + y \le 20$ (0,20) (4,0)

$x + y \le 12$ (0,12) (12,0)

$x + 3y \ge 18$ (0,6) (18,0)

1st quadrant $\begin{cases} x \ge 0 \\ y \ge 0 \end{cases}$



$5x + y \le 20$

$(0,0) \Rightarrow 5(0) + 0 \le 20$

$0 \le 20$ True

Shade toward (0,0)

$x + y \le 12$

$(0,0) \Rightarrow 0 + 0 \le 12$

$0 \le 12$ True. Shade toward (0,0)

Find corner points:

Can get two immediately: (0,6) and (0,12)

$x + 3y \ge 18$

$(0,0) \Rightarrow 0 + 3(0) \ge 18$

$0 \ge 18$ False. Shade away from (0,0)

Find Corner Point A:

$x + y = 12 \xrightarrow{(-1)} -x - y = -12$

$5x + y = 20 \longrightarrow 5x + y = 20$

$4x = 8$

$x = 2$

Put $x = 2$ into $x + y = 12$

$2 + y = 12$

$y = 10$

(2,10)

Find Corner Pt B

$x + 3y = 18 \longrightarrow x + 3y = 18$

$5x + y = 20 \xrightarrow{(-3)} -15x - 3y = -60$

$-14x = -42$

$x = 3$

Put $x = 3$ into $x + 3y = 18$: $3 + 3y = 18 \Rightarrow 3y = 15$

$y = 5$

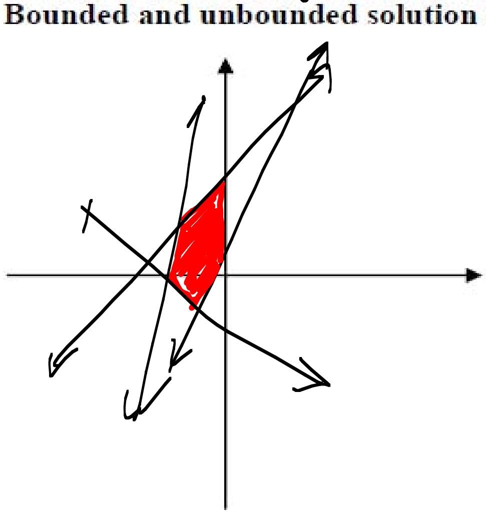
(3,5)

Corner Pts

(0,6) (3,5)

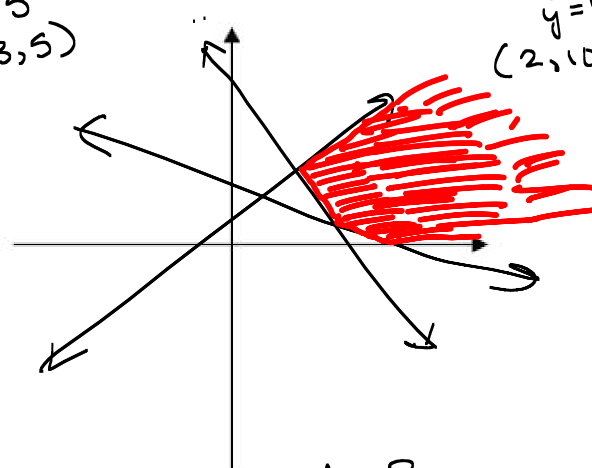
(0,12) (2,10)

Bounded and unbounded solution regions:



Bounded Region

(can be enclosed in a circle)



Unbounded Region

(Cannot be enclosed in a circle)

Example 3: Budget Cat Food Company supplies two distributors. One needs at least 200 boxes of cat food monthly, and the other needs at least 400. Budget Cat Food can make 800 boxes at the most. Write a system of inequalities to describe the situation and then graph the feasible region.