7.2: Sets

Definition: A *set* is a well-defined collection of objects. Each object in a set is called an *element* of that set.

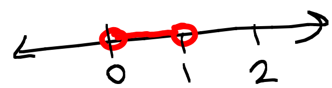
Examples of sets: All NBA players currently on NBA roster
All dogs registered with American Kennel Club
All U.S. citizens over 6 feet tall

Not sets: All people who play basketball
All cute dogs
All tall people Americans

Sets can be finite or infinite.

Examples of finite sets:
The set of NBA players
The integers between -100 and 100.
$-100, -99, -98, \dots, -3, -2, -1, 0, 1, 2, 3, \dots, 100$

Examples of infinite sets:
The set of all integers: $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
The set of real numbers between 0 and 1



Notation:

- We usually use capital letters for sets.
  We usually use lower-case letters for elements of a set.

$\in$ means "is an element of"

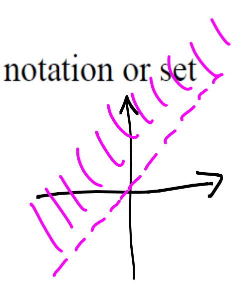
- $a \in A$ means a is an element of the set A. $a \in A$
  $a \notin A$ means a is not an element of the set A. $a \notin A$

$\notin$ means "is not an element of"

- The *empty set* is the set with no elements. It is denoted $\varnothing$. This is sometimes called the *null set*. $\varnothing$

- $S = \{x \mid P(x)\}$ means "S is the set of all x such that $P(x)$ is true". (called rule notation or set roster notation). (or set builder notation)
  "such that"

$\{(x, y) \mid y > x\}$



Example: $S = \{x \mid x \text{ is an even positive integer}\}$ means $S = \{2, 4, 6, 8, \dots\}$

Definition: We say two sets are *equal* if they have exactly the same elements.

Subsets:

Definition: If each element of a set A is also an element of set B, we say that A is a *subset* of B. This is denoted $A \subseteq B$ or $A \subset B$. If A is not a subset of B, we write $A \not\subset B$.

$A \subseteq B$ or $A \subset B$ $A \not\subset B$

Definition: We say A is a *proper subset* of B if $A \subseteq B$ but $A \neq B$. (In other words, every element of A is also an element of B, but B contains at least one element that is not in A.)

Note on notation: Some books use the symbol $\subset$ to indicate a proper subset. Some books use $\subset$ to indicate any subset, proper or not.

Definition: The set of all elements under consideration is called the *universal set*, (or universe) usually denoted U.

Example: If you're dealing with sets of real numbers, then U is the set of all real numbers. So "Wednesday" would not be an element of U, but 5.7 would be in U.

Example 1: Consider these sets.

$A = \{1,2,3,4,5,6\}$ $A = C$

$B = \{1,2,3,4,5,6,7,8\}$ $A \subseteq B$

$C = \{1,3,5,2,4,6\}$ $C \subseteq B$

A is a proper subset of B.
A is not a proper subset of C.
$A \subseteq C$
$C \subseteq A$

Note:

- $\varnothing$ is a subset of every set. (i.e. $\varnothing \subseteq A$ for every set A.)
- Every set is a subset of itself. (i.e. $A \subseteq A$ for every set A.)

Example 2: List all subsets of $\{1,2,3\}$.

$\{1,2,3\}, \{1,2\}, \{3,1\}, \{2,3\}, \{1\}, \{2\}, \{3\}, \varnothing$

Note: If a set has n elements, how many subsets does it have? 2^n subsets. So a set with 3 elements has $2^3 = 8$ subsets.

Set operations:

- Union $\cup$: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

  Key word: OR
- Intersection $\cap$: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

  Key word: AND
- Complement A' or A^C or $A^{\sim}$: $A' = \{x \in U \mid x \notin A\}$. Complement is A' or A^C or $A^{\sim}$

  Key word: NOT

Note: $A \subseteq (A \cup B)$ and $B \subseteq (A \cup B)$.

$(A \cap B) \subseteq A$ and $(A \cap B) \subseteq B$.

Definition: We say that A and B are *disjoint sets* if $A \cap B = \varnothing$.

Example 3: $U = \{1,2,3,4,5,6,7,8\}$

$H = \{1,3,5,7\}$

$K = \{1,3,5,6\}$

$J = \{2,4,6,8\}$

$L = \{2,3,4\}$

$H \cap K = \{1,3,5\}$

$K \cap L = \{3\}$

$H \cap J = \varnothing$ (H and J are disjoint... they do not overlap)

$H \cup K = \{1,3,5,7,6\} = \{1,3,5,6,7\}$

$L \cup K = \{2,3,4,1,5,6\} = \{1,2,3,4,5,6\}$

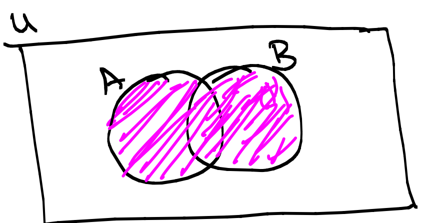
$L' = L^c =$ L complement.

$L' = \{1,5,6,7,8\}$

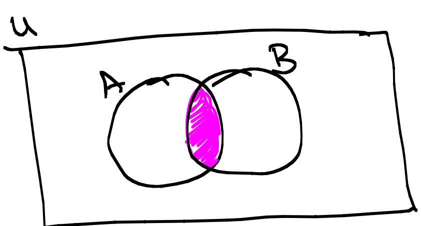
$K' = \{2,4,7,8\}$

Venn Diagrams: These help us visualize set relationships and operations.

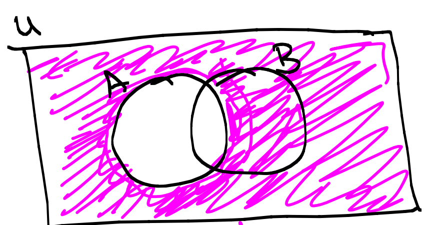
Example 4: Draw Venn diagrams for $A \cup B$, $A \cap B$, A', $(A \cap B)'$, $A' \cap B'$, and $A' \cap B$.



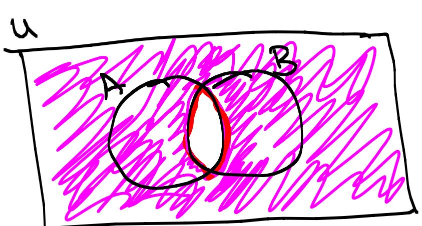
$A \cup B$



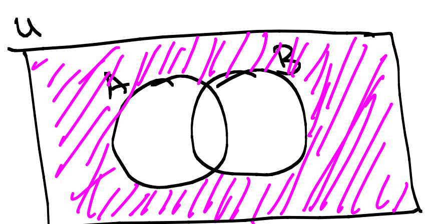
$A \cap B$



A'

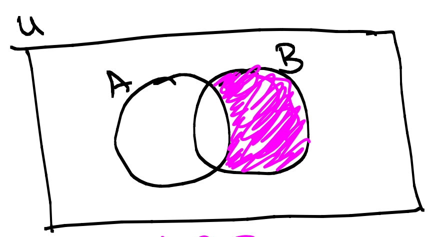


$(A \cap B)'$



$A' \cap B'$

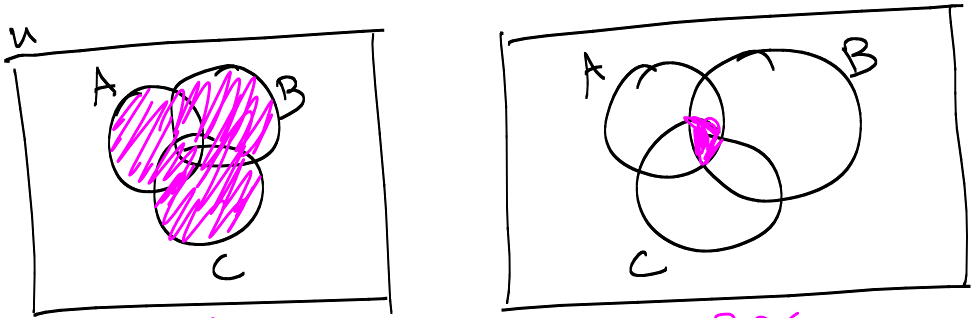
outside of A and outside of B



$A' \cap B$

outside of A and inside of B

Example 5: On a Venn diagram, shade $A \cup B \cup C$, $A \cap B \cap C$, and $(A \cup B') \cap C$.



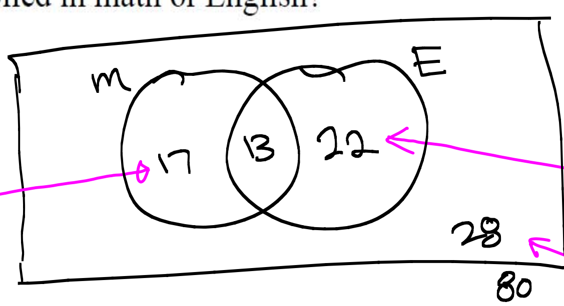
$A \cup B \cup C$ $\qquad$ $A \cap B \cap C$

Example 6: Consider a group of 80 students. 30 of them are enrolled in a math course and 35 are enrolled in an English course. 13 of the students are enrolled in an English course and also a math course. How many students are enrolled in math or English?

M: in math class

E: in English class

$30 - 13 = 17$



$35 - 13 = 22$

$80 - 17 - 13 - 22 = 28$

$$n(M \cup E) = 17 + 13 + 22 = 52$$

There are 52 students enrolled in math or English.

From Addition Principle

$n(M) = 30$

$n(E) = 35$

$n(M \cap E) = n(E \cap M) = 13$

$$n(M \cup E) = n(M) + n(E) - n(E \cap M) = 30 + 35 - 13 = 65 - 13 = \boxed{52}$$

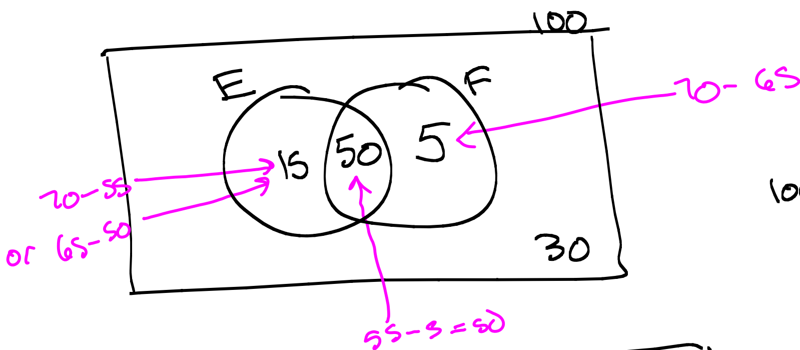
Notation: $n(A)$ means the number of elements in set A.

> Addition principle for Counting
>
> For any two sets A and B,
>
> $$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$
>
> If A and B are disjoint ($A \cap B = \varnothing$), then $n(A \cup B) = n(A) + n(B)$.

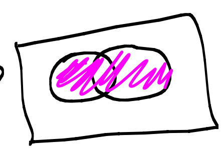
Example 7: 100 students are surveyed to determine if they had watched ESPN or Fox Sports Channel in the last 3 months. The results show that 65 students watched ESPN, 55 watched Fox Sports, and 30 watched neither.

a. How many people watched ESPN but not Fox Sports?
b. How many people watched Fox Sports but not ESPN?
c. How many watched both networks?

E: people who watched ESPN
F: people who watched Fox Sports



$100 - 30 = 70 \Rightarrow$



(a) $n(E \cap F') = \boxed{15}$

(b) $n(F \cap E') = \boxed{5}$

(c) $n(E \cap F) = \boxed{50}$

Example 8: I want to buy a car from Jay Austin Motors. Of all the cars on the lot, 89 cars have navigation systems, 100 have touch-screen controls, and 74 have blind spot alert systems. 32 cars have both navigation systems and blind spot alert, 40 have both a touch screen and a blind spot alert system, and 53 have a touch screen and a navigation system. Twelve cars have all three features, and 21 cars are base models with none of these features.

I strongly dislike having a touch screen, but I would like a navigation system and a blind spot alert system. How many cars do I have to choose from?

How many cars are on Jay Austin's lot?

See video!