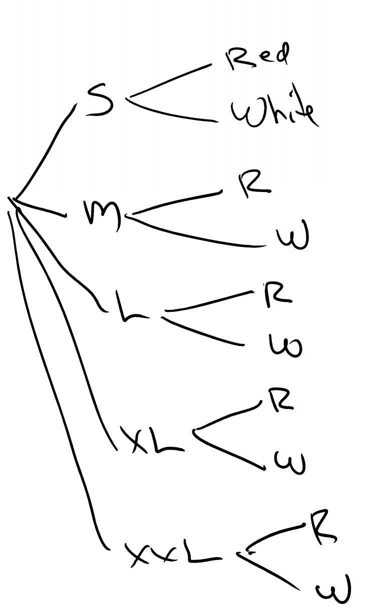
## 7.3: Basic Counting Principles

**Multiplication principle for counting:**

This principle is used to analyze sets which are determined by a sequence of operations.

**Example 1:** A company sells football jerseys. The jerseys come in sizes S, M, L, XL, and XXL. They also come in two colors: red for home games and white for away games. How many total types of jerseys does the company make?




RS
WS

RM

WM

$\underset{\text{5 sizes}}{5}\,\underset{\text{2 colors}}{(2)} = \boxed{\text{10 types of jerseys}}$

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Multiplication Principle (for counting)

Suppose that $n$ choices must be made, with

$m_1$ ways to make choice 1,
$m_2$ ways to make choice 2,
$m_3$ ways to make choice 3,
.
.
.
$m_n$ ways to make choice $n$.

Then there are $m_1 \cdot m_2 \cdot \ldots \cdot m_n$ ways to make the entire sequence of choices.

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Ex: Suppose a tent company offers 3 sizes of tents, each in 5 different colors. For each, you have the option of adding a reinforced raincover. How many types of tent are possible?  $3(5)(2) = \boxed{30}$



**Example 2:** How many license plate "numbers" can be formed by using a letter, followed by two digits, followed by three more letters?

$$\underset{\text{letter}}{26} \cdot \underset{\text{digit}}{10} \cdot \underset{\text{digit}}{10} \cdot \underset{\text{letter}}{26} \cdot \underset{\text{letter}}{26} \cdot \underset{\text{letter}}{26} = 10^2 \cdot 26^4 = \boxed{45\ 697\ 600}$$

How many can be formed assuming adjacent letters and numbers must be different?

$$26 \cdot 10 \cdot 9 \cdot 26 \cdot 25 \cdot 25$$

How many can be formed assuming letters and numbers cannot be repeated?

$$26 \cdot 10 \cdot 9 \cdot 25 \cdot 24 \cdot 23$$

Products like this occur so frequently that special counting formulas and notations have been developed for them. These formulas use a function called the factorial.

The Factorial:

For a natural number (positive integer) $n$, $n!$ is called "$n$-factorial". It is defined as follows:

$$n! = n(n-1)(n-2)\ldots(3)(2)(1)$$

$$n! = n(n-1)!$$

$$0! = 1$$ Note: $1! = 1$ also

**Example 3:**

$$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{720}$$

$$\frac{8!}{7!} = \frac{8 \cdot \cancel{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = \boxed{8}$$

$$\frac{97!}{3!94!} = \frac{97 \cdot 96 \cdot 95 \cdot \cancel{94!}}{3 \cdot 2 \cdot 1 \cdot \cancel{94!}} = \frac{97 \cdot 96 \cdot 95}{6} = \boxed{147\ 440}$$

Note: Factorials grow very rapidly!

**Example 4:** Compare 5!, 10!, and 15!.

$5! = 120$

$10! = 3\ 628\ 800$

$15! \approx 1.3077 \times 10^{12}$