Example 9: Solve the system using Gauss-Jordan elimination.

$$3y - z = -1$$
$$x + 5y - z = -4$$
$$-3x + 6y + 2z = 11$$

$$\begin{array}{cccc} x & y & z & \text{RHS (right-hand side)} \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 0 & 3 & -1 & -1 \\ 1 & 5 & -1 & -4 \\ -3 & 6 & 2 & 11 \end{array}\right]$$

Goal:
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & * \\ 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \end{array}\right]$$

$R_1 \leftrightarrow R_2$ $\sim$
$$\left[\begin{array}{cccc} 1 & 5 & -1 & -4 \\ 0 & 3 & -1 & -1 \\ -3 & 6 & 2 & 11 \end{array}\right]$$

want 0s

$\underset{3R_1 + R_3 \to R_3}{\sim}$
$$\left[\begin{array}{cccc} 1 & 5 & -1 & -4 \\ 0 & 3 & -1 & -1 \\ 0 & 21 & -1 & -1 \end{array}\right]$$

want 1

$\frac{1}{3}R_2 \to R_2$ $\sim$
$$\left[\begin{array}{cccc} 1 & 5 & -1 & -4 \\ 0 & 1 & -1/3 & -1/3 \\ 0 & 21 & -1 & -1 \end{array}\right]$$

want 0 want 0

$-5R_2 + R_1 \to R_1$ $\sim$ $-21R_2 + R_3 \to R_3$
$$\left[\begin{array}{cccc} 1 & 0 & 2/3 & -7/3 \\ 0 & 1 & -1/3 & -1/3 \\ 0 & 0 & 6 & 6 \end{array}\right]$$

want 1

$\frac{1}{6}R_3 \to R_3$
$$\left[\begin{array}{cccc} 1 & 0 & 2/3 & -7/3 \\ 0 & 1 & -1/3 & -1/3 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

want 0s

$-\frac{2}{3}R_3 + R_1 \to R_1$ $\sim$ $\frac{1}{3}R_3 + R_2 \to R_2$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

1st row:
$$1x + 0y + 0z = -3$$
R_2: $0x + 1y + 0z = 0$

R_3: $0x + 0y + 1z = 1$

$$\boxed{\begin{array}{c} x = -3 \\ y = 0 \\ z = 1 \end{array}}$$

Check if:
$$3y - z = -1$$
$$3(0) - 1 = -1 \checkmark$$

$$x + 5y - z = -4$$
$$-3 + 5(0) - 1 = -4 \checkmark$$

$$-3x + 6y + 2z = 11$$
$$-3(-3) + 6(0) + 2(1) = 11$$
$$9 + 0 + 2 = 11 \checkmark$$

Ex 11: Solve using Gauss-Jordan elimination

want 1

$$3x - y + 2z = 2 \qquad \begin{bmatrix} 3 & -1 & 2 & 2 \\ 2 & -2 & 5 & 7 \end{bmatrix}$$
$$2x - 2y + 5z = 7$$

$$\underset{\sim}{\tfrac{1}{3}R_1 \to R_1} \begin{bmatrix} 1 & -1/3 & 2/3 & 2/3 \\ 2 & -2 & 5 & 7 \end{bmatrix} \underset{\sim}{-2R_1 + R_2 \to R_2} \begin{bmatrix} 1 & -1/3 & 2/3 & 2/3 \\ 0 & -4/3 & 11/3 & 17/3 \end{bmatrix}$$

want 0 want 1

want 0

$$-\tfrac{3}{4}R_2 \to R_2 \begin{bmatrix} 1 & -1/3 & 2/3 & 2/3 \\ 0 & 1 & -11/4 & -17/4 \end{bmatrix} \underset{\sim}{\tfrac{1}{3}R_2 + R_1 \to R_1}$$

$$\begin{bmatrix} 1 & 0 & -1/4 & -3/4 \\ 0 & 1 & -11/4 & -17/4 \end{bmatrix} \qquad \begin{aligned} R_1&: 1x + 0y - \tfrac{1}{4}z = -\tfrac{3}{4} \\ R_2&: 0x + 1y - \tfrac{11}{4}z = -\tfrac{17}{4} \end{aligned}$$

Solve R_1 for x:

R_2

$$\boxed{\begin{aligned} x &= \tfrac{1}{4}z - \tfrac{3}{4} \\ y &= \tfrac{11}{4}z - \tfrac{17}{4} \\ z &= \text{any real number} \end{aligned}}$$

This is an example of a <u>dependent</u> system (infinitely many solutions).

Another way to write the solution:

Let $z = t$:

$$\boxed{\begin{aligned} x &= \tfrac{1}{4}t - \tfrac{3}{4} \\ y &= \tfrac{11}{4}t - \tfrac{17}{4} \\ z &= t \\ t &\text{ is any real number.} \end{aligned}}$$

Here, t is called a <u>parameter</u> (another variable that the main variables can be written in terms of.

Ex 12: Solve the system using Gauss-Jordan elimination. 4.2/4.3.8

$$-4x + 8y = 2$$
$$x - 3y = 4$$
$$2x + 2y = -3$$

$$\left[\begin{array}{cc|c} -4 & 8 & 2 \\ 1 & -3 & 4 \\ 2 & 2 & -3 \end{array}\right] \quad R_1 \longleftrightarrow R_2 \quad \sim$$

want 0s

want 1

$$\left[\begin{array}{ccc} 1 & -3 & 4 \\ -4 & 8 & 2 \\ 2 & 2 & -3 \end{array}\right] \underset{-2R_1+R_3 \to R_3}{\overset{4R_1+R_2 \to R_2}{\sim}} \left[\begin{array}{ccc} 1 & -3 & 4 \\ 0 & -4 & 18 \\ 0 & 8 & -11 \end{array}\right] \quad -\frac{1}{4}R_2 \longrightarrow R_2 \quad \sim$$

want 0s

$$\left[\begin{array}{ccc} 1 & -3 & 4 \\ 0 & 1 & -\frac{9}{2} \\ 0 & 8 & -11 \end{array}\right] \underset{-8R_2+R_3 \to R_3}{\overset{3R_2+R_1 \to R_1}{\sim}} \left[\begin{array}{ccc} 1 & 0 & -19/2 \\ 0 & 1 & -9/2 \\ 0 & 0 & 25 \end{array}\right]$$

R_1: $1x + 0y = -\frac{19}{2}$

R_2: $0x + 1y = -\frac{9}{2}$

R_3: $0x + 0y = 25$

$\Downarrow$

$0 = 25$ False! Impossible!

$\boxed{\text{No solution}}$ This system is <u>inconsistent</u> and has no solution.