1324-BZBS14e-notes-8-1-sample-spaces-events-prob-intro

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8.1: Sample Spaces, Events, and Probability

An *experiment* is an activity with observable results. An experiment that does not always give the same result, even under the same conditions, is called a *random experiment*. Repetitions of an experiment are called *trials*.

> Examples: rolling dice, drawing cards, counting the number of defective radios in a shipment, etc.

In probability theory, we'll usually use the word *experiment* to mean a *random experiment*.

For a given experiment, we can make a list of outcomes of the experiment, called *simple events*, such that in each trial, one and only one of the simple events will occur. The set of all such simple events is called the *sample space*.

Any subset of the sample space is called an *event*. If such a subset contains more than one element of the sample space, we call the subset a *compound event*. If an event $E = \varnothing$, we call the event E an *impossible event*. If S is the sample space and $E = S$, then E is a *certain event*.

Example 1: We roll a single six-sided die.

Sample Space: $S = \{1, 2, 3, 4, 5, 6\}$

Those are all the possible outcomes

Example 2: We spin a roulette wheel (a wheel that has the integers 0 to 36 on it).

$S = \{0, 1, 2, 3, \ldots, 34, 35, 36\}$

Example 3: The manager of a local cinema records the number of patrons attending a particular movie. The theater has a seating capacity of 500.

a. What is an appropriate sample space for this experiment?

$S = \{0, 1, 2, 3, \ldots, 499, 500\}$

b. Describe the event E that fewer than 50 people attend the movie?

$E = \{0, 1, 2, 3, \ldots, 49\}$

c. Describe the event F that the theater is more than half full at the movie?

$F = \{251, 252, 253, \ldots, 499, 500\}$

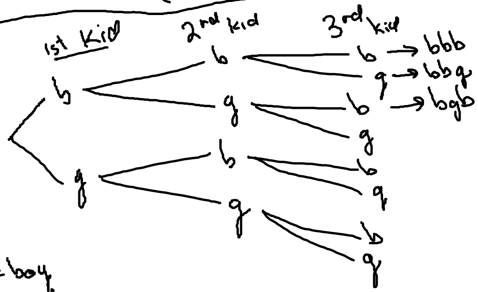
Example 4: An experiment consists of studying the composition ~~of~~ a three-child ~~family~~ families.

a. Describe an appropriate sample space for this experiment if we are interested in the genders of the children in the order of their births.
b. Describe the event E that there are two girls and a boy in the family.
c. Describe the event F that the oldest child is a girl.
d. Describe the event G that the oldest child is a girl and the youngest is a boy.
e. Describe an appropriate sample space if we are only interested in the number of girls and not the order.

$\boxed{\text{a) } S = \{bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg\}}$

Should be 8 possibilities because Multiplication principle:

$\underbrace{2}_{\text{1st child}} \cdot \underbrace{2}_{\text{2nd child}} \cdot \underbrace{2}_{\text{3rd child}} = 8$



b=boy
g=girl

b) $E = \{bgg, gbg, ggb\}$

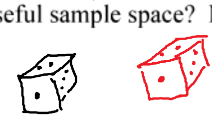
c) $F = \{ggg, ggb, gbg, gbb\}$

d) $G = \{ggb, gbb\}$

e) $S_2 = \{0, 1, 2, 3\}$

Example 5: Roll a pair of dice. What is the most useful sample space? Describe the following events:

a. The sum is 6.
b. The sum is 2.
c. The sum is more than 9.



we make ordered pairs:

Ex $(\underset{\text{black die}}{2}, \underset{\text{Red die}}{4})$

$(\underset{\text{black die}}{4}, \underset{\text{red die}}{2})$

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$
$(2,1)\quad (2,2)\quad (2,3), (2,4), (2,5), (2,6)$
$(3,1), (3,2) \ldots \quad (3,6)$
$(4,1), (4,2) \ldots \quad (4,6)$
$(5,1), (5,2) \ldots \quad (5,6)$
$(6,1)\quad (6,2) \ldots \quad (6,6)\}$

number of outcomes in sample space:

$n(S) = 6 \cdot 6 = 36$

a) Sum is 6: $\{(1,5), (2,4), (3,3), (4,2), (5,1)\}$

b) Sum is 2: $\{(1,1)\}$

c) Sum is more than 9: $\{(5,5), (5,6), (6,5), (6,6), (4,6), (6,4)\}$

Notice: 3 ways to get 10
2 ways to get 11
1 ways to get 12

Probability:

In many sample spaces, all the outcomes, or simple events, are equally likely to occur. In these cases, we use the *equally likely assumption*. If several choices are possible for an experiment's sample space, it is often best to choose one in which all outcomes are equally likely.

> Equally likely assumption:
>
> If all events in a sample space are equally likely to occur, the probability of each is $\frac{1}{n}$, where n is the number of simple events in the sample space.

The equally likely assumption results in a basic principle of probability.

> Basic Probability Principle
>
> Let S be a sample space of equally likely outcomes, and let the event E be a subset of S. Then the probability that event E occurs is
>
> $$P(E) = \frac{n(E)}{n(S)}.$$

heads, tails equally likely

Example 6: Flip two fair coins. What is the probability of getting
- a. Two tails?
- b. At least one head?
- c. Exactly one head?
- d. Two heads or two tails?
- e. Three tails?

$S = \{HH, HT, TH, TT\}$

$n(S) = 4$ all equally likely

H: heads
T: tails

(a) Two tails
$A = \{TT\}$
$n(A) = 1$
$P(A) = P(\text{2 tails}) = \frac{n(A)}{n(S)} = \boxed{\frac{1}{4}}$

(b) at least 1 head
$B = \{HH, HT, TH\}$
$P(B) = \frac{n(B)}{n(S)} = \boxed{\frac{3}{4}}$

(c) Exactly 1 head
$C = \{HT, TH\}$
$P(C) = \frac{n(C)}{n(S)} = \frac{2}{4} = \boxed{\frac{1}{2}}$

(d) 2 heads or 2 tails
$D = \{HH, TT\}$ $n(D) = 2$
$P(D) = \frac{n(D)}{n(S)} = \frac{2}{4} = \boxed{\frac{1}{2}}$

(e) 3 tails
$E = \emptyset$ not possible with only 2 coins!
$P(E) = 0$ This is an impossible event.

(f) Find Probability of getting less than 3 tails:
$F = \{HH, TH, HT, TT\}$
$P(F) = \frac{4}{4} = \boxed{1}$ This is a certain event.

Important Note: For any event E,

$$0 \le P(E) \le 1.$$

If $P(E) = 0$, then E is an *impossible event*.
If $P(E) = 1$, then E is an *certain event*.

Example 7: Draw a single card from a standard card deck. What is the probability that it is
a. A spade?
b. A queen?
c. Black?
d. Higher than a 7? (assume aces high.)

S = set of all possible cards that can be drawn

$n(S) = 52$ all are equally likely

b) Drawing a Queen

$Q = \{Q\heartsuit, Q\diamondsuit, Q\clubsuit, Q\spadesuit\}$

$P(Q) = \frac{n(Q)}{n(S)} = \frac{4}{52} = \frac{2}{26} = \boxed{\frac{1}{13}}$

B = black cards

$n(B) = 13 + 13 = 26$

$P(B) = \frac{n(B)}{n(S)} = \frac{26}{52} = \boxed{\frac{1}{2}}$

Drawing a spade

Spades = {A, 2, ... 10, J, Q, K of spades}

$P(\text{Spades}) = \frac{n(\text{Spades})}{n(S)} = \frac{13}{52} = \boxed{\frac{1}{4}}$

d) $D = \{8, 9, 10, J, Q, K, A$ of each suit$\}$

$n(D) = 7(4) = 28$

4 suits

$P(D) = \frac{28}{52} = \frac{14}{26} = \boxed{\frac{7}{13}}$

Example 8: Roll a single die. What is the probability of rolling

a) a 5?
b) a prime number?
c) a multiple of 3?
d) a number larger than 10?
e) a number smaller than 10?

fair: each outcome is equally likely to happen.

Example 9: Roll two fair dice. What is the probability that the sum is 7? That the sum is less than 6?

$S = \{(1,1), (1,2), \dots, (1,6)$
$(2,1) \dots (2,6)$
$\vdots \quad \vdots$
$(6,1) \dots (6,6)\}$

Set of 36 ordered pairs. $n(S) = 36$

Each ordered pair is equally likely.

Probability of getting $(1,5)$ is $\frac{1}{36}$.

Prob. of getting $(2,3)$ is $\frac{1}{36}$.

Prob of getting $(6,6)$ is $\frac{1}{36}$.

(a) What is the probability their sum is 7?

$A = \{(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)\}$

$n(A) = 6$

$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \boxed{\frac{1}{6}}$

(b) What is prob that the sum is less than 6?

$B = \{\underbrace{(1,1)}_{\text{sum 2}}, \underbrace{(1,2), (2,1)}_{\text{sum is 3}}, \underbrace{(1,3), (3,1), (2,2)}_{\text{sum is 4}}, \underbrace{(1,4), (4,1), (2,3), (3,2)}_{\text{sum is 5}}\}$

$P(B) = \frac{n(B)}{n(S)} = \frac{10}{36} = \boxed{\frac{5}{18}}$

Example 10: Jennifer's electricity went out last night. Not having a flashlight, she had to choose a T-shirt at random in the dark. She owns 10 white T-shirts, 5 gray ones, 6 black ones and 2 red ones. What is the probability that she chose a gray T-shirt?

S = set of all possible T-shirts

$n(S) = 10 + 5 + 6 + 2 = 23$

$P(\text{Gray}) = \frac{n(\text{Gray})}{n(S)} = \boxed{\frac{5}{23}}$

Example 11: Suppose that Joe, Steve, Suzy, and Lisa work for the same company. The company wants to send two representatives to a particular conference and needs two to stay home and take care of the customers. All of them want to attend the conference, so they decide to put their names in a hat and draw two at random. What is the probability that Suzy and Lisa are selected?

S = all the different ways 2 can be chosen to go

$S = \{\text{JoSt}, \text{JoSu}, \text{JoLi}, \text{StSu}, \text{StLi}, \text{SuLi}\}$

$n(S) = 6$

$E = \{\text{SuLi}\}$

$P(E) = \frac{n(E)}{n(S)} = \boxed{\frac{1}{6}}$

OR $n(S) = C_{4,2} = 6$

$E = \{\text{SuLi}\}$

$n(E) = 1$

$P(E) = \boxed{\frac{1}{6}}$

Example 12: The tuition categories for North Harris College students in the spring semester of 2007 are given in the table below.

In-District	9,794
Out-of-District	428
Out-of-State	104
International	250
Other	9

n = 10 585

a) What is the probability that a randomly selected student lives in-district?

$\frac{9794}{10585} \approx \boxed{0.92527} \Rightarrow$ About 92.5% of students are in-district.

b) What is the probability that a randomly selected student is an international student?

$\frac{250}{10585} \approx \boxed{0.024}$ So about 2.4% are international students

Example 13: The age distribution of Spring 2007 North Harris College students in the spring semester of 2007 is given in the table below.

Age of Students	
Under 20	33.4%
20-24	35.6%
25-29	11.3%
30-39	11.5%
40-49	6.3%
50 and Over	2.0%
Total	100%

less than 30

at least 40

What is the probability that a randomly selected student is

a) under 20 years old?

$\boxed{0.334}$

Prob of Over 50: $\boxed{0.02}$

b) at least ~~Over~~ 40? $0.063 + 0.02 = \boxed{0.083}$

Less than 30: $0.334 + 0.356 + 0.113 = \boxed{0.803}$

c) Over 60? not enough info to get this exactly

$\leq 2.0\%$

$P(\text{Age} > 60) \leq 0.02$

Probability assignments:

Suppose $S = \{e_1, e_2, \dots e_n\}$ is a sample space. It contains *n* simple events, or outcomes. To each outcome, we can assign a number $P(e_i)$, called the *probability of the event* e_i.

Rules for probabilities:

- The probability of a simple event is a number between 0 and 1, inclusive. In other words, $0 \le P(e_i) \le 1$.
- The probabilities of all the simple events in the sample space add up to 1. In other words, $P(e_1) + P(e_2) + \dots + P(e_n) = 1$.

Any probability assignment that meets these conditions is called an *acceptable probability assignment.*

A probability assignment that reflects the actual or expected percentage of times a simple event occurs is called *reasonable*. In other words, it is reasonable if it makes sense based on the real world.

Example 14: Roll a single die.

$P(\{1\}) = \frac{1}{6}$, $P(\{2\}) = \frac{1}{6}$

Shorthand: $P(1) = \frac{1}{6}$

$P(2) = \frac{1}{6}$, etc.

Probability of an event E:

Given a sample space *S* and an acceptable probability assignment, then for any event *E* ($E \subset S$), the following rules apply:

- If $E = \varnothing$, then $P(E) = 0$.
- If *E* is a simple event, then $P(E)$ is given by the original probability assignment.
- If *E* is a compound event, then $P(E)$ is the sum of the probabilities of all the simple events in *E*.
- If $E = S$, then $P(E) = P(S) = 1$.

Steps for finding $P(E)$:

1. Set up an appropriate sample space.
2. Assign acceptable probabilities to each simple event.
3. $P(E)$ is the sum of the probabilities of all the simple events in *E*.

There are two basic methods of assigning probabilities.

- Theoretical: no experiments are conducted—use assumptions and reasoning.
- Empirical: use results of actual experiments. (This is also called *relative frequency probability*.)

Empirical (relative frequency) probability approximation:

$$P(E) \approx \frac{\text{frequency of occurence of } E}{\text{total number of trials}} = \frac{f(E)}{n}$$

(The larger n is, the better the approximation.)

Example 15: Suppose that we survey 134 students at Lone Star College-North Harris and find that 97 of them own an iPhone. Use this information to estimate the probability that a randomly selected LSC-NH student owns an iPhone. What is the probability that a randomly selected student does not own an iPhone?

Probability problem vs "how many?"

Example 16: Draw 5 cards from a standard 52-card deck.
a. What is the $\boxed{\text{probability}}$ of getting 5 spades?
b. What is the probability of getting 2 kings, 2 queens, and a jack?

S = Sample space: set of all possible 5-card hands

$n(S) = C_{52,5} = 2\ 598\ 960$. This will be our denominator
All 2,598,960 hands are equally likely

(a) 5 spades

A = set of all 5-card hands with 5 spades

$n(A) = C_{13,5} =$

$P(A) = \frac{n(A)}{n(S)} = \frac{1287}{2\ 598\ 960} \approx \boxed{0.00495}$ So you will get all spades about % of the time.

(b) 2 kings, 2 queens, 1 jack

$n(B) = \underbrace{C_{4,2}}_{2K} \cdot \underbrace{C_{4,2}}_{2Q} \cdot \underbrace{C_{4,1}}_{1J} = C_{4,2} \cdot C_{4,2} \cdot 4 = 6 \cdot 6 \cdot 4$

$P(B) = \frac{n(B)}{n(S)} = \frac{6 \cdot 6 \cdot 4}{2\ 598\ 960} \approx \boxed{5.5407 \times 10^{-5}} = \boxed{0.000055407}$

Example 17: Joe and Susan Thomas work in a department with 12 other people. Four employees are to be randomly chosen to attend an important conference in the Bahamas. What is the probability that both Joe and Susan will be chosen?

S: set of all different ways to choose a subset of 4 from a group of 14 people

$n(S) = C_{14,4}$

E: The group of 4 includes Joe & Susan

$n(E) = \underbrace{1}_{\text{Joe}} \cdot \underbrace{1}_{\text{Susan}} \underbrace{C_{12,2}}_{\text{2 other people}}$

$P(E) = \frac{1 \cdot 1 \cdot C_{12,2}}{C_{14,4}} = \frac{66}{1001} \approx \boxed{0.0659}$

(So about a "6% chance")

Ex 16 c: What is the probability of getting at least 3 spades? (in a 5-card hand)

$n(C) = \underbrace{C_{13,3}}_{\text{3 spades}} \cdot \underbrace{C_{39,2}}_{\text{2 non spades}} + \underbrace{C_{13,4}}_{\text{4 spades}} \cdot \underbrace{39}_{\text{1 non spade}} + \underbrace{C_{13,5}}_{\text{5 spades}}$

$P(C) = \frac{C_{13,3} \cdot C_{39,2} + C_{13,4} \cdot 39 + C_{13,5}}{C_{52,5}}$

$= \frac{286 \cdot 741 + 715 \cdot 39 + 1287}{2\ 598\ 960}$

$= \frac{241\ 098}{2\ 598\ 960} \approx \boxed{0.0927}$

(So this happens about 9% of the time)

Example 18: Suppose that you had written six thank-you cards and addressed six envelopes. While you were away from your desk, your young child, attempting to be helpful, put the cards into the envelopes randomly, stamped and mailed them.

a) What is the probability that all the cards were put into the correct envelopes?
b) What is the probability that only two cards were sent to the wrong people?

S = set of all possible ways to arrange 6 cards into 6 envelopes

$$\underset{1^{st}\text{ envelope}}{\underline{6}} \cdot \underset{2^{nd}\text{ env}}{\underline{5}} \cdot \underset{3^{rd}}{\underline{4}} \cdot \underset{4^{th}}{\underline{3}} \cdot \underset{5^{th}}{\underline{2}} \cdot \underset{6^{th}}{\underline{1}} = 6!$$

$n(S) = 6! = 720$

(a) How many ways to get all cards in correct envelopes? 1

Prob all are correct is $\boxed{\frac{1}{720}} \approx 0.00139$

(b) 4 correct, 2 are wrong
How many ways to choose 4 of the 6 envelopes to be correct? $C_{6,4}$

$n(B) = \underbrace{C_{6,4}}_{4\text{ env correct}} \cdot \underbrace{1}_{1\text{ way for those to be wrong}} = 15$ So $P(B) = \frac{15}{720} \approx \boxed{0.02083}$

Example 19: Suppose that a shipment of 100 radios contains three that are defective. If a quality-control engineer selects a random sample of 5 radios, what is the probability that the sample contains at least one defective radio?

3 bad radios in shipment

97 good radios

S: all possible 5-radio samples

$n(S) = C_{100,5}$

$n(E) = \underbrace{3}_{1\text{ bad radio}} \cdot \underbrace{C_{97,4}}_{4\text{ good radios}} + \underbrace{C_{3,2}}_{2\text{ bad radios}} \cdot \underbrace{C_{97,3}}_{3\text{ good}} + \underbrace{1}_{3\text{ bad radios}} \cdot \underbrace{C_{97,2}}_{2\text{ good}}$

$$P(E) = \frac{3 \cdot C_{97,4} + 3 \cdot C_{97,3} + 1 \cdot C_{97,2}}{C_{100,5}}$$

Example 20: My tack locker has a combination lock with a wheel that must be turned to 3 numbers in succession. The wheel contains the numbers 0 through 39, and such locks do not involve repeated numbers. What is the probability of guessing the correct combination?

S = set of all different sets of 3 numbers in order

$n(S) = P_{40,3} = 59\,280$

E: the lock opens (only 1 way for this to happen!)

$n(E) = 1$

$\boxed{P(E) = \frac{1}{59\,280}}$

Example 21: In the Lotto Texas game, players choose six numbers from the numbers 1-54, without repeating numbers. The player wins the big jackpot if he chooses all six winning numbers, regardless of order. He can win a smaller 2nd, 3rd or 4th prize by matching 5, 4, or 3 of the six winning numbers.

What is the probability he wins the jackpot?

What is the probability he wins 2nd prize?

3rd prize?

4th prize?