1324-BZBS14e-notes-8-2-unions-intersects-complements-odds

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8.2: Union, Intersection, and Complement of Events; Odds

Unions and Intersections:

Example 1: Roll a single die.

a. What is the probability of rolling a number that is even and divisible by 3?

b. What is the probability of rolling a number that is even or divisible by 3?

$S = \{1,2,3,4,5,6\}$

E: Even numbers: 2,4,6

D: Divisible by 3: 3,6

$E = \{2,4,6\}$

$D = \{3,6\}$

(a) $A = \{6\}$

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{6}$

(b) $B = E \cup D = \{2,4,6,3\}$

$P(B) = \frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

Recall:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

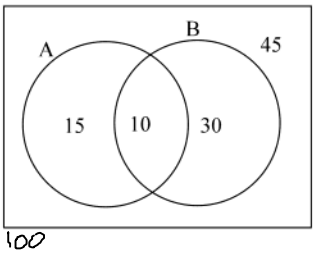
Probability of a Union of Two Events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If the two events are mutually exclusive (disjoint):

$$P(A \cup B) = P(A) + P(B)$$

Example 2: Assume that an equally likely sample space is described by the Venn diagram below.



$P(A \cup B) = \frac{15+10+30}{100} = \frac{55}{100} = 0.55$

$P(A \cap B) = \frac{10}{100} = 0.10$

$P(A) = \frac{15+10}{100} = \frac{25}{100} = 0.25$

Ex 2½: Suppose $P(A) = 0.31$, $P(B) = 0.52$, and $P(A \cap B) = 0.17$. Find $P(A \cup B)$.

$P(A \cup B) = \frac{14+17+35}{100} = \frac{66}{100} = 0.66$

Complements:

Probability of a complement:
$P(E') = 1 - P(E)$
$P(E) = 1 - P(E')$

E-complement: E^c or E'

Example 3: Suppose that the probability of someone voting for a certain candidate is 0.46. What is the probability of not voting for the candidate?

$1 - 0.46 = \boxed{0.54}$

Example 4: Roll a pair of dice. What is the probability of rolling a sum of 4 or more?

S = set of 36 ordered pairs $\{(1,1), (1,2) \ldots, (1,6)$ ... $(6,1)$ $(6,6)\}$

A: Sum is 4 or more

Sum is 4, 5, 6, 7, 8, 9, 10, 11, 12

A': Sum is 2 or 3

$A' = \{(1,1), (1,2), (2,1)\}$

$P(A') = \frac{3}{36}$

$P(A) = 1 - P(A') = 1 - \frac{3}{36} = \frac{33}{36} = \boxed{\frac{11}{12}}$

Odds:

Sometimes the likelihood (or unlikelihood) of an event is described using *odds* instead of probabilities.

Summary:

Probability: The event is contrasted against the whole.

Odds: The event is contrasted against the complement.

Converting from probability to odds:

From Probability to Odds:

- Odds for $E = \dfrac{P(E)}{P(E')}$
- Odds against $E = \dfrac{P(E')}{P(E)}$

When possible, express odds as ratios of whole numbers.

Example 5: Roll a pair of dice. What are the odds for rolling a sum of 3? What are the odds against rolling a sum of 3?

$n(S) = 36$

E: sum is 3: {(2,1),(1,2)}

so $n(E') = 34$

Odds for E: $\frac{2}{34} = \frac{1}{17}$ or 1:17

Odds against E: $\frac{34}{2} = \frac{17}{1}$ or 17:1

Example 6: What are the odds against ~~rolling~~ drawing an ace when drawing a single card from a standard deck?

A: Aces

$n(A) = 4$

$n(A') = 48$

48 non-aces

odds against getting an ace are $\frac{48}{4} = \frac{12}{1}$ or 12:1

odds for getting an ace: $\frac{4}{48} = \frac{1}{12}$ or 1:12

Example 7: Suppose that, based upon genetics, a child has a 0.08 probability of developing a certain disease. What are the odds against the child developing the disease?

D: gets disease

$P(D) = 0.08$

$P(D') = 0.92$

odds against disease: $\frac{0.92}{0.08} = \frac{92}{8} = \frac{23}{2}$ or 23:2

or 11.5 to 1

$\frac{23}{2} = 11.5 = \frac{11.5}{1}$

Converting odds to probability:

From Odds to Probability:
If odds for an event E are $\frac{m}{n}$, (i.e. m:n) then $P(E)=\frac{m}{m+n}$.

Example 8: If the odds against a horse winning a race are 7:1, what is the probability that the horse will win?

odds against winning are 7 : 1

(chances he loses (doesn't win) : chances he wins)

"Total chances": $7+1=8$

$P(\text{win}) = \frac{\text{chances he wins}}{\text{total chances}} = \boxed{\frac{1}{8}}$

Example 9: Suppose an insurance company has used past flood data to determine that determined that the odds against a particular house flooding are 150:1. What is the probability that the house floods?

150 : 1

(chances it doesn't flood : chances it floods)

Total chances: $150+1=151$

Prob. it floods is $\boxed{\frac{1}{151}}$