$1324\hbox{-}BZBS14e_Notes\hbox{-}3\hbox{-}2\hbox{-}compound\hbox{-}interest$

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(200-, \$1000(1.02)

3.2: Compound Interest

If at the end of a payment period, the interest due is reinvested at the same rate, then the interest as well as the principal will earn interest. This is called *compound interest*. The interest is paid into the account at the end of each compounding period. > A times per year

Example 1: Suppose you invest \$1000 compounded quarterly at an annual interest rate of 8%.

How much money will you have after one year?

P= \$1000) r=0.08, t= + 4 A= 8(1400 (1+0.08(4)) = \$1020 P= \$1026, 4=0.08, t= 1 yr A= \$1020 (1+0.08(4)) = \$1020 (1.02) = \$1040.40 P= \$1040.40, r=0.08, f= 7 A = \$1040.40 (1.02)

S= \$1001.31 <u>></u> 44 Av∙ F= \$1001.31 (1.02) You

Q1: \$ 1000 (1.02) Q4: \$ 1000 (1.02)

Compound Interest:

~ \$1061.2i

$$A = P(1+i)^{n}$$

$$= P\left(1 + \frac{r}{m}\right)^{n}$$

$$= P\left(1 + \frac{r}{m}\right)^{mi}$$

t= time in years

where

 $i = \frac{r}{m}$ is the interest rate per compounding period

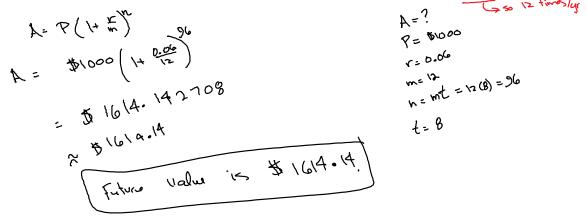
r = annual interest rate

m = number of compounding periods per year

 $n = \text{total number of compounding periods} (n = \sqrt{n})$

P = principal (present value)

A = amount (future value) at the end of n compounding periods.



Example 3: How much should I invest now at 4% interest compounded monthly in order to have \$10,000 in 6 years?

$$A = P(H \frac{r}{m})$$

$$A = \frac{10000}{P} = P(H \frac{0.04}{12})^{72}$$

$$7869.418714 = P$$

$$1869.418714 = P$$

$$1869.42$$

$$1869.43$$

$$4 = 6$$

Example 4: You decide to invest some money so that you will have \$1,000,000 on your 75th birthday. At 8% compounded quarterly, how much should you invest on your 25th birthday?

birthday. At 8% compounded quarterly, how much should you invest on your 25th birthday?

$$A = P(1 + \frac{r}{m})^{h}$$

$$A = \frac{1}{4} \cos 000$$

$$P = \frac{1}{4$$

Example 5: How long will it take \$5,000 to grow to \$7,000 if it is invested at 8% compounded

monthly? In
$$A = P(H = m)$$
 $A = P(H = m)$
 $A = P(H = m)$

Example 6: How long will it take money to double if it is invested at 7.5% compounded

monthly?

$$A = P(H \text{ m}) \qquad P = ? \text{ (bod we don't core what } P = ? \text{ (bod we don't$$

Continuous compound interest:

In calculus, a fundamental topic is the *limit*, or limiting value of a function. If we allow the number of compounding periods per year to increase toward infinity, the amount A approaches the limiting value $A = Pe^{rt}$. The number e is a constant, $e \approx 2.71828$. The number e is irrational—it cannot be written as a fraction of integers, or as a decimal that ends or repeats.

e can be defined as the limiting value of $\left(1+\frac{1}{x}\right)^x$ as x approaches ∞ .

Start with the compound interest formula:

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

Substitute $x = \frac{m}{r}$ and then rearrange/simplify:

$$A = P \left[\left(1 + \frac{1}{x} \right)^x \right]^{rt}$$

As $x \to \infty$, $\left(1 + \frac{1}{x}\right)^x \to e$. This gives us the formula for continuous compound interest.

Continuous Compound Interest:

If principal P is compounded continuously at the annual interest rate r, then the amount at the end of t years is

$$A = Pe^{rt}$$
.

Example 7: How much must be invested now to have \$60,000 available in 10 years, if it is invested at 7% compounded (a) monthly? (b) continuously?

invested at 7% compounded (a) monthly? (b) continuously?

(a) monthly
$$A = P(1+P) \qquad P = P(1+P)$$

Mode:
$$l_N(e^x) = \chi$$

also $e^{l_N \chi} = \chi$

because $f(x) = e^x$ and $g(x) = l_N(\chi)$ ours inverses

3.2.5

Example 8: How long will it take \$5,000 to grow to \$7,000 if it is invested at 8% compounded continuously?

continuously?

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 $9.$

The effective rate, sometimes called the annual percentage yield, converts a compound interest rate to an equivalent simple interest rate. This allows us to compare interest rates which have different compounding periods.

Annual Percentage Yield (APY):

The annual percentage yield (APY), or effective rate, is given by

$$APY = r_e = \left(1 + \frac{r}{m}\right)^m - 1,$$

where

r = annual interest rate

m = number of compounding periods per year.

For interest compounded continuously, the APY is

$$APY = r_e = e^r - 1$$
.

					3.2.6	
Example 9: quarterly?	What is the annual	percentage yield	for money invest	ed at 6% compou	ınded	
Example 10: 9.2% compour	Which investment	is better, Note A	at 9% compound	ed monthly or No	ote B at	