1324-BZBS14e_Notes-3-4-present-value-annuity-with-amortization-schedules

Thursday, July 25, 2019 7:11 AM

1324-BZBS14e_Notes-3-4-present-value-annuity-with-am...

3.4: Present Value of an Annuity; Amortization

Present Value of an Ordinary Annuity

$$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$$

where

$PMT =$ periodic payment (made at end of period)

$i = \frac{r}{m} =$ rate per period

$n =$ number of payments (periods)

$PV =$ present value of all payments

Example 1: How much should you deposit into an account that pays 6% compounded semiannually so that $1,000 may be withdrawn every 6 months for three years?

The *amortization* of a debt is the process of paying it off in equal installments. For example, if I buy a new car and don't have the cash for it, I *amortize* the debt by making equal monthly payments.

Example 2: Suppose you want to finance an $800 television. The electronics store is willing to finance it for 18 months at 18% compounded monthly.

a. What are the monthly payments?
b. How much total interest will you pay?

Example 3: I buy a car for $20,000. I put $800 down and the dealer gives me $1800 for my trade-in. I finance the rest at 5.5% for five years (compounded monthly). What are my monthly payments? How much total money do I pay for the car? How much interest?

Total borrowed = $20000 − 1800 − 800 = $17 400 This is the present value

$$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$\$17\,400 = PMT\left[\frac{1-\left(1+\frac{0.055}{12}\right)^{-60}}{\left(\frac{0.055}{12}\right)}\right]$$

$PV = \$17\,400$
$PMT = ?$
$i = \frac{r}{m} = \frac{0.055}{12}$
$r = 0.055$
$m = 12$
$n = mt = 12(5) = 60$
$t = 5$

PMT = $332.36

Total of Payments: $332.36(60) = $19941.6

Total paid: Payments + Down Payment + Trade-in
= $19941.6 + $800 + $1800 = $22541.6

Interest paid = total payments − amount financed
= $19941.60 − $17400 = $2541.60

I will pay $2541.60 in interest. The total amount paid for the car is $22541.60

Example 4: Scott and Jennifer are considering buying a house. The house they like costs ~~$110,000~~ $135000 in 2003; and they have saved ~~$20,000~~ for a down payment. The bank required a 3% down payment

a. What will be their monthly payment for a 30-year loan at ~~8%~~ 6% (compounded monthly)? How much interest will they pay?

b. What will be their monthly payment for a 15-year loan at ~~8%~~ 6% (compounded monthly)? How much interest will they pay?

c. What will be their monthly payment for a 15-year loan at 4.6% (compounded monthly)? How much interest will they pay?

3% down payment is how much?
$0.03(\$135000) = \$4050$

(a) 30-year mortgage:

$$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$PV = \$135000 - \$4050 = \$130\,950$
$PMT = ?$
$i = \frac{r}{m} = \frac{0.06}{12}$
$n = mt = 12(30) = 360$
$t = 30$
$m = 12$

$$\$130\,950 = PMT\left[\frac{1-\left(1+\frac{0.06}{12}\right)^{-360}}{\left(\frac{0.06}{12}\right)}\right]$$

$PMT = \$785.11$

Total of payments: $360(\$785.11) = \$282\,639.60$
Interest paid: $\$282\,639.60 - \$130\,950$
$= \$151\,689.60$

Interest paid on 30-yr mortgage: $151 689.60

(b) 15-year mortgage:

$$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$n = mt = 12(15) = 180$
$t = 15$
everything else stays the same

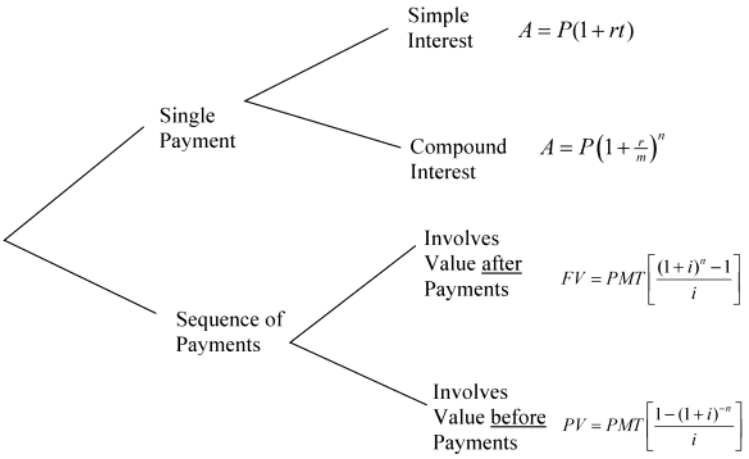
$$\$130\,950 = PMT\left[\frac{1-\left(1+\frac{0.06}{12}\right)^{-180}}{\left(\frac{0.06}{12}\right)}\right]$$

$PMT = \$1105.03$

Total of payments $= 180(\$1105.03) = \$198\,905.40$
Interest paid: $\$198\,905.40 - \$130\,950$
$= \$67\,955.40$

Interest paid on 15-year mortgage: $67 955.40

Summary of formulas:



Amortization schedules:

Example 5: Suppose that Scott and Jennifer decided to buy the ~~$140,000~~ 135000 house with the ~~$10,000~~ $40,500 down payment, financed for 30 years at ~~8%~~ 6%. How much of their first 3 payments went to interest? How much would have gone toward interest had they opted for the 15-year loan at 5%?

$i = \frac{r}{m} = \frac{0.06}{12} = 0.005$

Pmt = $785.11

30-year mortgage.

1st month: Interest = i(principal) = 0.005($130 950) = $654.75

Month 1 Pmt: $785.11 = $654.75 (Interest) + $130.36 (Principal)

2nd month: Principal: $130 950 − $130.36 = $130 819.64

Interest: 0.005($130 819.64) = $654.10

Month 2 Pmt: $785.11 = $654.10 (Interest) + $131.01 (Principal)

Month 3: Principal: 130 819.64 − 131.01 = $130 688.63

Interest: 0.005($130 688.63) = $653.44

Month 3: $785.11 = $653.44 (Interest) + $131.67 (Principal)

in 2003

Example 6: Scott and Jennifer ended buying a $135,000 house. They chose the 30-year mortgage at 6% annual interest rate, with a 3% down payment. Six years later, they were able to refinance at 3.875% with no closing costs. This time, they chose a 15-year mortgage. (re-fi in 2009)

a. What was their remaining balance when they refinanced?
b. How much interest did they pay during the first six years (on the original mortgage)?
c. How much interest would they have paid during the remaining 24 years of the original mortgage?
d. What are the payments on the new mortgage?
e. Suppose that after paying on the new mortgage for nine years (with no additional principal payments), the house is worth $200,000. How much equity do they have?

(a) After 6 years, we have made $6(12) = 72$ payments

There are $24(12) = 288$ payments remaining.

As far as bank is concerned, we now have a 24-year mortgage with payments of $785.11.

How big a loan can I get if I plan to make $785.11 payments for 24 years?

$$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$PV = \$785.11\left[\frac{1-\left(1+\frac{0.06}{12}\right)^{-288}}{\left(\frac{0.06}{12}\right)}\right]$$

PV = remaining balance = ?

PMT = $785.11

r = 0.06

m = 12

$i = \frac{r}{m} = \frac{0.06}{12}$

$n = mt = 12(24) = 288$

$PV = \$119\,685.40$

Remaining balance after 6 years is $119 685.40

How much did we pay in payments those 6 yrs? $72(\$785.11)$

$= \$56\,527.92$

(originally borrowed $130 950, so only decreased balance by $130 950 - $119 685.40

$= \$11\,264.60$

Suppose at this time (6 years after original loan, house is worth $140,000.

What is our equity? $140 000 - $119 685.40 = $20 314.60 equity after 6 years