1324-BZBS14e_Notes-4-4-basic-matrix-operations

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4.4: Matrices-Basic Operations

We will learn how to add matrices and how to multiply a matrix by a number (scalar).

Equality:

Two matrices are *equal* if they are the same size *and* all the corresponding elements are equal.

Example 1:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix},$$

$E \neq F$

$B \neq E$

$$D = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \; E = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \; F = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$B = D$

$G = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ $\quad D \neq G$

Addition:

Matrices can be added only if they are the same size. In this case, we add the corresponding elements in the matrices.

Example 2:

$$\begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} + \begin{bmatrix} 1 \\ 7 \end{bmatrix} \neq \boxed{\text{not possible}}$$

$$\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} 3 & 6 \\ 7 & 6 \end{bmatrix}}$$

$$\underbrace{\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}}_{3\times1} + \underbrace{\begin{bmatrix} 4 \\ -5 \\ 6 \end{bmatrix}}_{3\times1} = \underbrace{\boxed{\begin{bmatrix} 3 \\ -3 \\ 9 \end{bmatrix}}}_{3\times1}$$

Addition of numbers is associative and commutative. Addition of matrices is associative and commutative also.

- Commutative: $A + B = B + A$
- Associative: $(A + B) + C = A + (B + C)$

with numbers: commutative $\Rightarrow$ $2+3=3+2$ [both equal 5]

associative $\Rightarrow$ $(2+3)+4 = 2+(3+4)$ [both equal 9]

A *zero matrix* is a matrix with zero in all positions. The following are zero matrices of different sizes:

$$\underset{2\times 2}{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}} \qquad \underset{3\times 2}{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}$$

The *negative of a matrix A*, denoted $-A$, is the matrix with all elements that are the opposites of the corresponding elements in the matrix *A*.

Example 3:

$$A = \begin{bmatrix} -2 & 3 \\ 4 & -6 \end{bmatrix} \qquad -A = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}$$

Subtraction:

As with addition, subtraction can be performed only if matrices are the same size. The difference $A-B$ is defined to be $A+(-B)$. So to subtract, we just subtract the corresponding elements.

Example 4:

$$\begin{bmatrix} 1 & 2 & -6 \\ -3 & 4 & 5 \end{bmatrix} - \begin{bmatrix} 0 & -2 & -2 \\ 4 & 6 & 5 \end{bmatrix} = \begin{bmatrix} 1-0 & 2-(-2) & -6-(-2) \\ -3-4 & 4-6 & 5-5 \end{bmatrix} = \begin{bmatrix} 1 & 2+2 & -6+2 \\ -7 & -2 & 0 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 1 & 4 & -4 \\ -7 & -2 & 0 \end{bmatrix}}$$

2×3

Multiplication of a matrix by a number:

The product of a number *k* and a matrix *M*, denoted by *kM*, is the matrix formed by multiplying each element of *M* by *k*. This is often called *scalar multiplication*.

Example 5:

$$4\begin{bmatrix} 1 & 3 \\ -2 & 7 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 4 & 12 \\ -8 & 28 \\ 0 & -16 \end{bmatrix}$$

Product of a row matrix and a column matrix (in that order):

The product of a $1\times n$ row matrix A and an $n\times 1$ column matrix is the 1×1 matrix given by

$$AB = \begin{bmatrix} a_1 & a_2 \cdots a_n \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} a_1b_1 + a_2b_2 + \cdots + a_nb_n \end{bmatrix}.$$

<u>Note</u>: For this formula to hold, they must be in this order: row$\times$column. If they are in the other order (column$\times$row), you get a different result. We'll see one like this later.

<u>Note</u>: The number of elements in the row and column must be the same in order for the multiplication to be defined.

<u>**Example 6:**</u>

$$\begin{bmatrix} 1 & -2 & 3 & -5 \end{bmatrix} \begin{bmatrix} -4 \\ 0 \\ 2 \\ -4 \end{bmatrix} = 1(-4) - 2(0) + 3(2) - 5(-4) = -4 - 0 + 6 + 20 = \boxed{22}$$

$$\underset{1\times 3}{\begin{bmatrix} 21 & -32 & 19 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 11 \\ 23 \\ 13 \end{bmatrix}} = 21(11) - 32(23) + 19(13) = \boxed{-258}$$

$$\underset{1\times 4}{\begin{bmatrix} 3 & -2 & 5 & 3 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}} = \boxed{\text{not possible}}$$

sizes don't work

Matrix multiplication:

If A is an $m\times p$ and B is a $p\times n$ matrix, then the product of these is denoted AB and it is an $m\times n$ matrix.

$m\times p \quad p\times n$ must match!

The entries in the matrix AB are formed as follows: the element in the ith row and jth column is the product of the ith row of A with the jth column of B.

<u>Important Note</u>: If the number of columns of A is not equal to the number of rows of B, the product AB is not defined! The matrices cannot be multiplied!!

Example 7:

$$A=\begin{bmatrix}2 & 1 & 3\\ 0 & -2 & -1\end{bmatrix},\ B=\begin{bmatrix}1 & 0\\ 4 & -2\\ 1 & 1\end{bmatrix} \Rightarrow 2\times 2$$

$$2\times\cancel{3} \qquad \underset{=}{\cancel{3}}\times 2$$

R_0C_0

$$AB=\begin{bmatrix}2 & 1 & 3\\ 0 & -2 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ 4 & -2\\ 1 & 1\end{bmatrix}=\begin{bmatrix}\boxed{9} & \boxed{\ }\\ \boxed{\ } & \boxed{\ }\end{bmatrix}$$

$R_0 \times C_0$

$R_0C_0: 2(1)+1(4)+3(1)$

$= 2+4+3 = 9$

$$AB=\begin{bmatrix}2 & 1 & 3\\ 0 & -2 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ 4 & -2\\ 1 & 1\end{bmatrix}=\begin{bmatrix}9 & \boxed{1}\\ \boxed{\ } & \boxed{\ }\end{bmatrix}$$

$R_0 \times C_1$

$2(0)+1(-2)+3(1)$

$= 0-2+3 = 1$

$$AB=\begin{bmatrix}2 & 1 & 3\\ 0 & -2 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ 4 & -2\\ 1 & 1\end{bmatrix}=\begin{bmatrix}9 & 1\\ \underset{R_1C_0}{\boxed{-9}} & \boxed{\ }\end{bmatrix}$$

$R_1 \times C_0$

$0(1)-2(4)-1(1)$

$= 0-8-1 = -9$

$$AB=\begin{bmatrix}2 & 1 & 3\\ 0 & -2 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ 4 & -2\\ 1 & 1\end{bmatrix}=\boxed{\begin{bmatrix}9 & 1\\ -9 & \boxed{3}\end{bmatrix}}$$

$R_1 \times C_1$

$0(0)-2(-2)-1(1)$

$= 0+4-1 = 3$

Example 8:

$$\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}\begin{bmatrix}3 & 2\\ -1 & 4\end{bmatrix}=\begin{bmatrix}1 & 10\\ 5 & 8\end{bmatrix}$$

Example 9:

$$\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}4 & 5 & 6\end{bmatrix} = \begin{bmatrix}4 & 5 & 6\\8 & 10 & 12\\12 & 15 & 18\end{bmatrix}$$

$3\times\cancel{1}\quad\cancel{1}\times 3 \Rightarrow 3\times 3$

match

Example 10:

$$\begin{bmatrix}-1 & 3\\4 & 2\\-2 & 5\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\6 & 1 & -2\end{bmatrix} = \begin{bmatrix}17 & 1 & -9\\16 & 10 & 8\\28 & 1 & -16\end{bmatrix}$$

$3\times 2\quad 2\times 3$

match

$3\times\cancel{2}\quad\cancel{2}\times 3 \Rightarrow 3\times 3$

Example 11:

$$\begin{bmatrix}1 & 3 & 1\\2 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} =$$

$\boxed{\text{not possible}}$

$2\times 3\quad 2\times 3$

$\neq$

Example 12:

$$\begin{bmatrix}1 & 2\\3 & 6\end{bmatrix}\begin{bmatrix}2 & 6\\-1 & -3\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

So Zero Product Property does not hold for matrices compare with Ex 12

Example 13:

$$\begin{bmatrix} 2 & 6 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 20 & 40 \\ -10 & -20 \end{bmatrix}$$

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Note: Ex 12 and Ex 13 are the same matrices, multiplied in different orders, with different results. So $AB \neq BA$, in general.

Example 14:

$$\begin{bmatrix} 2 & -4 & 3 \\ -3 & 1 & 5 \\ 10 & -2 & 7 \end{bmatrix}^2$$

$$= \begin{bmatrix} 2 & -4 & 3 \\ -3 & 1 & 5 \\ 10 & -2 & 7 \end{bmatrix}\begin{bmatrix} 2 & -4 & 3 \\ -3 & 1 & 5 \\ 10 & -2 & 7 \end{bmatrix} = \begin{bmatrix} 46 & -18 & 7 \\ 4 & 3 & 31 \\ 96 & -56 & 69 \end{bmatrix}$$