# 1324-BZBS14e_Notes-6-3-dual-simplex

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1324-BZBS14e_Notes-6-3-dual-simplex



## 6.3: The Dual Problem: Minimization with $\geq$ Problem Constraints

The simplex method can be modified to solve minimization problems.

**The transpose of a matrix:**

The transpose of a matrix $A$ is called $A^T$ and is formed by interchanging the rows and columns of $A$.

**Example 1:** Find the transpose of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$. $(3 \times 2)$

$$A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \qquad (2 \times 3)$$

**The dual problem:**

Every minimization problem with $\geq$ constraints can be associated with a maximization problem with $\leq$ constraints. This maximization problem is called the *dual problem*.

**Example 1:** Minimize $C = 2y_1 + y_2$

Standard minimization Problem

Subject to
$$y_1 + y_2 \geq 8$$
$$y_1 + 2y_2 \geq 4$$
$$y_1 \geq 0$$
$$y_2 \geq 0$$

First, we create a matrix $A$ using the constraints and the objective function, with the objective function on the bottom row:

$$A = \begin{bmatrix} 1 & 1 & 8 \\ 1 & 2 & 4 \\ 2 & 1 & * \end{bmatrix}$$

Next, form the transpose $A^T$:

$$A^T = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 8 & 4 & * \end{bmatrix}$$

From the transpose, write a new linear programming problem with new variables:

note these are ≤ now

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 8 & 4 & \# \end{bmatrix} \Longrightarrow \begin{aligned} x_1 + x_2 &\le 2 \\ x_1 + 2x_2 &\le 1 \\ 8x_1 + 4x_2 &= Z \end{aligned}$$

All variables must be new

The dual problem is:

Know how to write the dual problem

$$\boxed{\begin{aligned} \text{Maximize} \quad Z &= 8x_1 + 4x_2 \\ x_1 + x_2 &\le 2 \\ x_1 + 2x_2 &\le 1 \\ x_1 &\ge 0 \\ x_2 &\ge 0 \end{aligned}}$$

The dual problem is a standard maximization problem

(can do Simplex on it)

<u>Theorem of Duality</u>:

The objective function *w* of a minimizing linear programming problem takes on a minimum value if and only if the objective function *z* of the corresponding dual maximizing problem takes on a maximum value. The maximum value of *z* is equal to the minimum value of *w*.

So, after forming the dual problem, use the simplex method to solve it.

- For slack variables, use the variables of the original minimization problem.
- When writing the solution, the values of the original variables are read from the bottom row.

$$\begin{aligned} x_1 + x_2 + y_1 \qquad\qquad &= 2 \\ x_1 + 2x_2 \qquad + y_2 \qquad &= 1 \\ -8x_1 - 4x_2 \qquad\qquad + z &= 0 \end{aligned}$$

$$\begin{array}{cccccc} x_1 & x_2 & y_1 & y_2 & z & RHS \\ \end{array}$$
$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 2 \\ 1 & 2 & 0 & 1 & 0 & 1 \\ -8 & -4 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{matrix} \frac{2}{1} = 2 \\ \frac{1}{1} = 1 \end{matrix}$$

← smallest

most negative

Do the pivot:

$-1R_1 + R_0 \to R_0$
$8R_1 + R_2 \to R_2$

$$\begin{array}{cccccc} x_1 & x_2 & y_1 & y_2 & z & RHS \\ \end{array}$$
$$\begin{bmatrix} 0 & -1 & 1 & -1 & 0 & 1 \\ 1 & 2 & 0 & 1 & 0 & 1 \\ 0 & 12 & \underbrace{0}_{y_1} & \underbrace{8}_{y_2} & 1 & 8 \end{bmatrix}$$

read values of $y_1$ and $y_2$ in <u>original</u> problem

<u>Solution to Dual Problem:</u>

max is $Z=8$, reached when $x_1 = 1$ and $x_2 = 0$

<u>Solution to Original Minimization Problem</u>

min is $C=8$, reached when $y_1 = 0$ and $y_2 = 8$

**Example 2:** (Example from Section 5.3—plant food)

Minimize $C = 30x_1 + 35x_2$

Subject to
$$\begin{aligned} 20x_1 + 10x_2 &\geq 460 \\ 30x_1 + 30x_2 &\geq 960 \\ 5x_1 + 10x_2 &\geq 220 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \end{aligned}$$

**Example 3:** An oil company operates two refineries in a certain city. Refinery I has an output of 200, 100, and 100 barrels of low-, medium-, and high-grade oil per day, respectively. Refinery II has an output of 100, 200, and 600 barrels of low-, medium-, and high-grade oil per day, respectively. The company wishes to produce at least 1000, 1400, and 3000 barrels of low-, medium-, and high-grade oil to fill an order. If it costs \$20,000/day to operate refinery I and \$30,000/day to operate refinery II, determine how many days each refinery should be operated to meet the requirements of the order at minimum cost to the company.