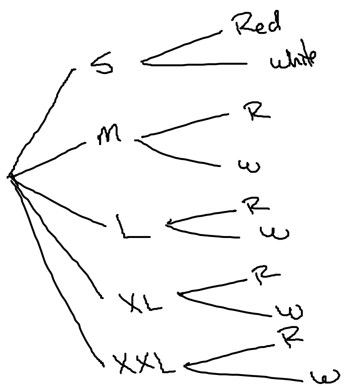
1324-BZBS14e_Notes-7-3-basic-counting-principles

7.3: Basic Counting Principles

Multiplication principle for counting:

This principle is used to analyze sets which are determined by a sequence of operations.

Example 1: A company sells football jerseys. The jerseys come in sizes S, M, L, XL, and XXL. They also come in two colors: red for home games and white for away games. How many total types of jerseys does the company make?



There are $5\,(2) = \boxed{10 \text{ jerseys}}$

5 size options

2 color options

Multiplication Principle

Suppose that n choices must be made, with

m_1 ways to make choice 1,
m_2 ways to make choice 2,
m_3 ways to make choice 3,
.
.
.
m_n ways to make choice n.

Then there are $m_1 \cdot m_2 \cdot \ldots \cdot m_n$ ways to make the entire sequence of choices.

Example 2: How many license plate "numbers" can be formed by using a letter, followed by two digits, followed by three more letters?

$\underset{\text{letter}}{26} \cdot \underset{\text{digit}}{10} \cdot \underset{\text{digit}}{10} \cdot \underset{\text{letter}}{26} \cdot \underset{\text{letter}}{26} \cdot \underset{\text{letter}}{26} = 10^2 \cdot 26^4 = 45\ 697\ 600$

How many can be formed assuming adjacent letters and numbers must be different?

$26 \cdot 10 \cdot 9 \cdot 26 \cdot 25 \cdot 25$

How many can be formed assuming letters and numbers cannot be repeated?

$26 \cdot 10 \cdot 9 \cdot 25 \cdot 24 \cdot 23 = 10 \cdot 9 \cdot 26 \cdot 25 \cdot 24 \cdot 23$

Products like this occur so frequently that special counting formulas and notations have been developed for them. These formulas use a function called the factorial.

The Factorial:

For a natural number (positive integer) n, $n!$ is called "n-factorial". It is defined as follows:

$n! = n(n-1)(n-2)\ldots(3)(2)(1)$

$n! = n(n-1)!$ Ex: $53! = 53 \cdot 52! = 53 \cdot 52 \cdot 51!$

$0! = 1$ ← by definition

$1! = 1$

Example 3:

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{720}$

$\frac{8!}{7!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{8 \cdot \cancel{7!}}{\cancel{7!}} = \frac{8}{1} = \boxed{8}$

$\frac{97!}{3!94!} = \frac{97 \cdot 96 \cdot 95 \cdot 94 \cdot 93 \cdot 92 \cdot \ldots 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1) 94 \cdot 93 \cdot 92 \cdot \ldots 3 \cdot 2 \cdot 1} = \frac{97 \cdot 96 \cdot 95 \cdot \cancel{94!}}{(3 \cdot 2 \cdot 1)\cancel{94!}} = \frac{97 \cdot 96 \cdot 95}{6} = \boxed{147\ 440}$

Note: Factorials grow very rapidly!

Example 4: Compare 5!, 10!, and 15!.

$5! = 120$

$10! = 3\ 628\ 800$

$15! = 1.307674368E12$

$= 1.307674368 \times 10^{12}$

$= 1.307674368000000 \times 10^{12}$

$= 1\ 307\ 674\ 368\ 000$