1324-BZBS14e-Notes-5-2-linear-inequalities

Thursday, July 18, 2019 1:22 AM

1324-BZBS14e-Notes-...

5.2: Systems of Linear Inequalities in Two Variables

Solving systems of linear inequalities graphically:

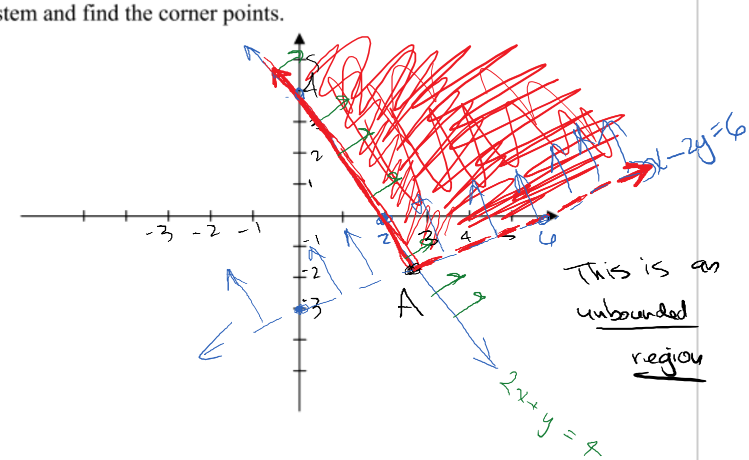
Now we'll solve systems of several inequalities.

We want to find the graph of all order pairs (x, y) that simultaneously satisfy all the inequalities in the system. The graph is called the *solution region*, or the *feasible region*, for the system. To find the solution region, we graph each inequality in the system (this will give a shaded area for each). The area that is included in *all* of them is the feasible region.

A *corner point* of a feasible region is a point in the solution region that is the intersection of two boundary lines.

Example 1: Solve the following system and find the corner points.

$$x - 2y < 6$$
$$2x + y \geq 4$$



This is an unbounded region

1st graph the lines:

$x - 2y = 6$ (0,-3) (6,0)

$2x + y = 4$ (0,4) (2,0)

Use intercepts to graph the lines.

$x - 2y < 6$

Test pt: $(0,0) \Rightarrow 0 - 2(0) < 6$

$0 < 6$ true

Shade half containing (0,0)

$2x + y \geq 4$

Test pt $(0,0) \Rightarrow 2(0) + 0 \geq 4$

$0 \geq 4$ False!

Shade half not containing (0,0)

Find corner point A:

$$\begin{aligned} x - 2y &= 6 \\ 2x + y &= 4 \end{aligned} \xrightarrow{(\times 2)} \begin{aligned} x - 2y &= 6 \\ 4x + 2y &= 8 \\ \hline 5x &= 14 \end{aligned}$$

$x = \frac{14}{5}$

$x = 2\frac{4}{5}$

$$\begin{aligned} x - 2y &= 6 \xrightarrow{-2} -2x + 4y = -12 \\ 2x + y &= 4 \longrightarrow \quad 2x + y = 4 \end{aligned}$$

$5y = -8$

$y = -\frac{8}{5}$

$y = -1\frac{3}{5}$

Corner point: $\boxed{\left(2\frac{4}{5}, -1\frac{3}{5}\right)}$

or $\boxed{(2.8, -1.6)}$

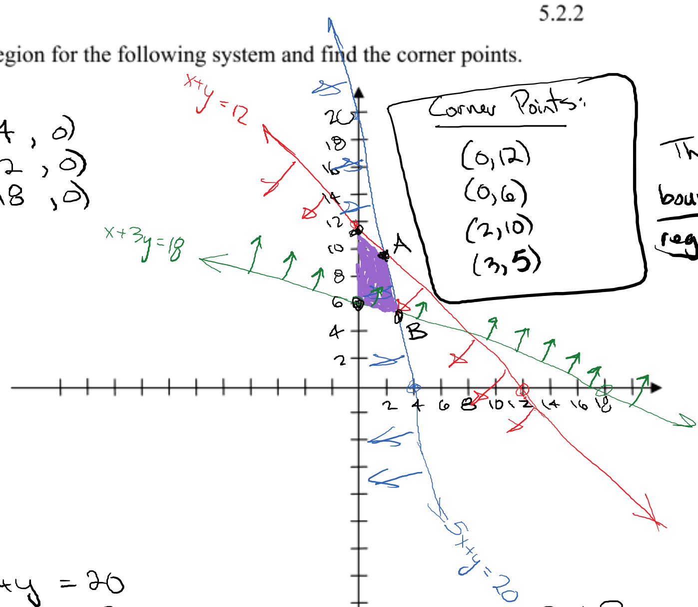
Example 2: Graph the feasible region for the following system and find the corner points.

$5x + y \le 20$ (0,20) (4 , 0)

$x + y \le 12$ (0,12)(12 , 0)

$x + 3y \ge 18$ (0,6)(18 ,0)

nonnegative constraints (restrict us to Quadrant I) $\begin{cases} x \ge 0 \\ y \ge 0 \end{cases}$



Corner Points:
(0,12)
(0,6)
(2,10)
(3,5)

This is a bounded region

$x + 3y \geqslant 18$

$(0,0) \Rightarrow 0 + 3(0) \geqslant 18$

$0 \geqslant 18$ false

Find Corner Point A:

$5x + y = 20 \longrightarrow 5x + y = 20$

$x + y = 12 \xrightarrow{(-1)} -x - y = -12$

Add: $4x = 8$

$x = 2$

Put $x = 2$ into $x + y = 12$:

$2 + y = 12$

$y = 10$

(2,10)

Find Corner Point B:

$x + 3y = 18 \longrightarrow x + 3y = 18$

$5x + y = 20 \xrightarrow{(-3)} -15x - 3y = -60$

$-14x = -42$

$x = \frac{-42}{-14} = 3$

Put $x = 3$ into $x + 3y = 18$

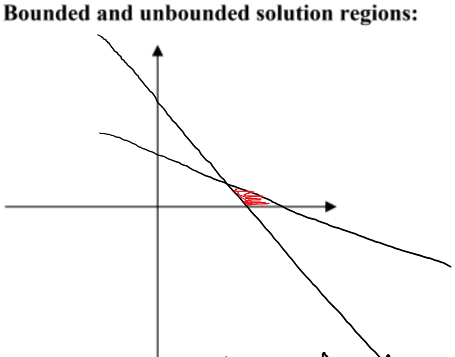
$3 + 3y = 18$

$3y = 15$

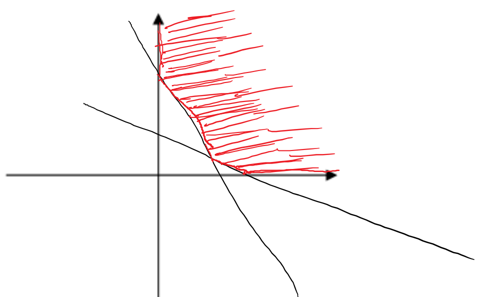
$y = 5$

(3, 5)

Bounded and unbounded solution regions:



Bounded Region: can be enclosed in a circle



Unbounded Region: cannot be enclosed in a circle

Example 3: Budget Cat Food Company supplies two distributors. One needs at least 200 boxes of cat food monthly, and the other needs at least 400. Budget Cat Food can make 800 boxes at the most. Write a system of inequalities to describe the situation and then graph the feasible region.