1314-2-2-Notes-more-on-functions

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2.2: More on Functions and Their Graphs

The difference quotient:

The expression $\frac{f(x+h)-f(x)}{h}$ is called a *difference quotient* for *f*. The difference quotient is important in our understanding of the rate of change of a function.

Example 1: For the function $f(x)=7-3x$, find and simplify the difference quotient.

$$\frac{f(x+h)-f(x)}{h} = \frac{\overbrace{7-3(x+h)}^{f(x+h)} - \overbrace{(7-3x)}^{f(x)}}{h} = \frac{\cancel{7}-3\cancel{x}-3h-\cancel{7}+\cancel{3x}}{h}$$

$$= \frac{-3\cancel{h}^{1}}{\cancel{h}_{1}} = \frac{-3}{1} = \boxed{-3}$$

Example 2: For the function $f(x)=x^2+2x-1$, find and simplify the difference quotient.

$$\frac{f(x+h)-f(x)}{h} = \frac{\overbrace{(x+h)^2+2(x+h)-1}^{f(x+h)} - \overbrace{(x^2+2x-1)}^{f(x)}}{h}$$

$$= \frac{\cancel{x^2}+2hx+h^2+\cancel{2x}+2h-\cancel{1}-\cancel{x^2}-\cancel{2x}+\cancel{1}}{h}$$

$$= \frac{2hx+h^2+2h}{h} = \frac{\cancel{h}^{1}(2x+h+2)}{\cancel{h}_{1}} = \boxed{2x+h+2}$$

Piecewise Defined Functions:
(Functions defined in "pieces")

Example 3: $f(x)=\begin{cases}3-x, & \text{if } x\le 1\\ x^2, & \text{if } x>1\end{cases}$

There are two different rules. The rule we use depends on which x we put in.

Calculate $f(4),\ f(-5),\ f(1)$.

To find $f(4)$, note that $4>1$ so we use $f(x)=x^2$

$f(4) = 4^2 = \boxed{16}$

To find $f(-5)$, note that $-5\le 1$, so we use $f(x)=3-x$

$f(-5)=3-(-5)$
$= 3+5 = \boxed{8}$

To find $f(1)$, note that $1\le 1$ so use $f(x)=3-x$

$f(1)=3-1=\boxed{2}$

Example 4: Write $f(x)=|x|$ as a piecewise function.

$$f(x)=|x|=\begin{cases}x & \text{if } x\ge 0\\ -x & \text{if } x<0\end{cases}$$

Example 5: $f(x)=\begin{cases}3x+4, & \text{if } x<-2\\ 7 & \text{if } x=-2\\ x^2+1, & \text{if } x>-2\end{cases}$

Calculate $f(3),\ f(-3),\ f(-2)$.

$f(3) = (3)^2+1 = 9+1 = \boxed{10}$

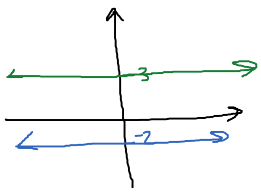
$f(-3) = 3(-3)+4 = -9+4 = \boxed{-5}$

$f(-2) = \boxed{7}$

Graphing piecewise-defined functions: (Piece functions)

Example 1: $f(x)=\begin{cases} 3 & \text{if } x \le 1 \\ -2 & \text{if } x > 1 \end{cases}$

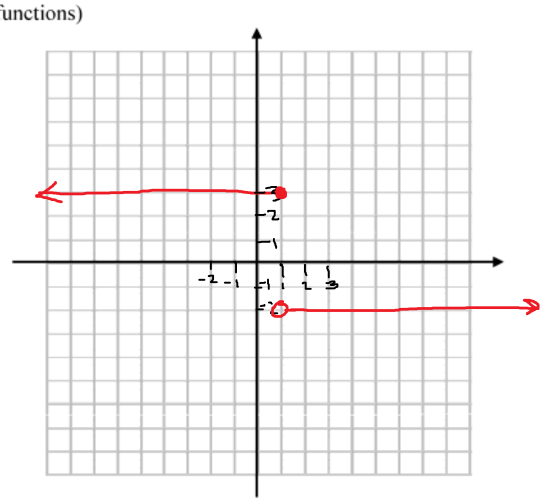
Graph each piece in its entirety, then erase the parts you don't need.



1^{st} graph $y=3$

also graph $y=-2$

left of 1: $y=3$
right of 1: $y=-2$

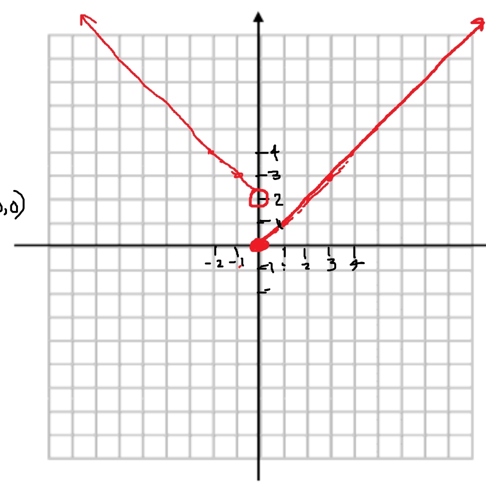


Example 2: $f(x)=\begin{cases} x & \text{if } x \ge 0 \\ -x+2 & \text{if } x < 0 \end{cases}$

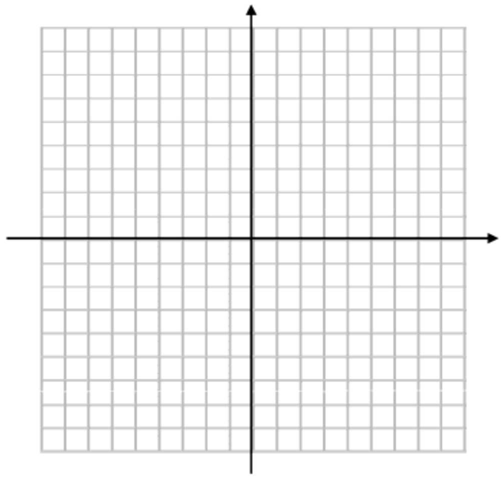
right of 0: $y=x$
$y=1x+0$
$m=1=+\frac{1}{1}$
$b=0 \Rightarrow$ y-intercept is $(0,0)$

Recall: $y=mx+b$ form: $m=$ slope
b is the y-intercept

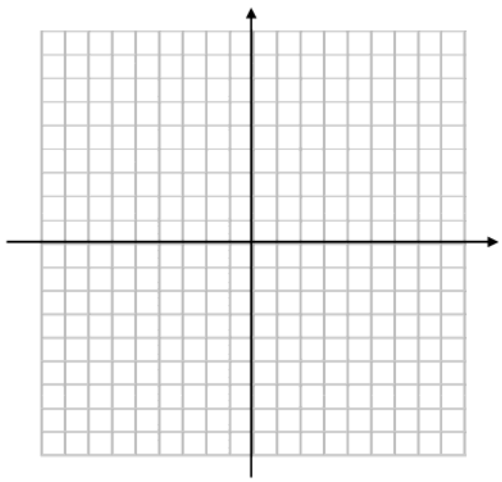
left of 0: $y=-x+2$
$y=-1x+2$
$m=-1=-\frac{1}{1}$
$b=2 \Rightarrow$ y-intercept is $(0,2)$



Example 3: $g(x)=\begin{cases}-3 & \text{if } x>2\\ x^2 & \text{if } x<2\\ 1 & \text{if } x=2\end{cases}$



Example 4: $f(x)=\begin{cases}x^3 & \text{if } x\geq 0\\ 3x+4 & \text{if } x<0\end{cases}$



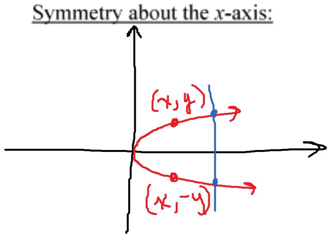
Symmetry:

A graph is *symmetric with respect to the x-axis* if, for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph. To check for x-axis symmetry, replace y by −y and see if it's equivalent to original

A graph is *symmetric with respect to the y-axis* if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph. To Check for y-axis symmetry, replace x by −x. See if it matches the original.

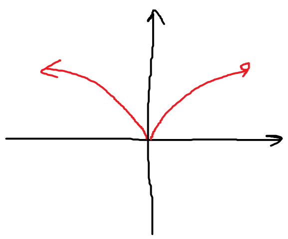
A graph is *symmetric with respect to the origin* if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph. To check for origin symmetry, replace x by −x and y by −y. See if it matches the original.

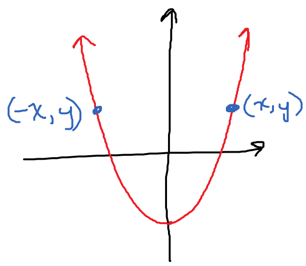
Symmetry about the *x*-axis:



This graph is symmetric about the x-axis. It fails the vertical line test, so it is not a function

Symmetry about the *y*-axis:

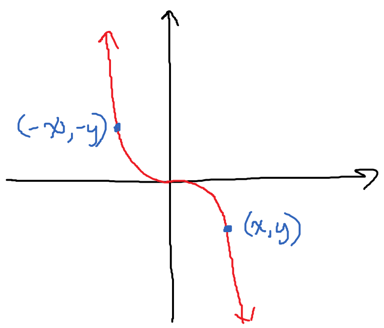


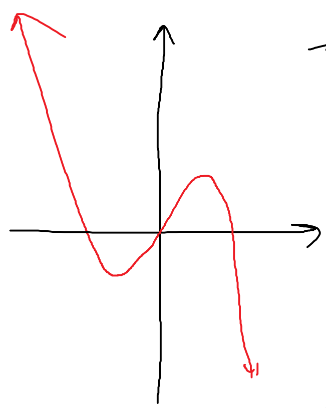


These are symmetric about the y-axis

Both of these are functions

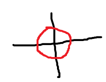
Symmetry about the origin:





These two functions are symmetric around the origin

Note:



symmetric around the origin, y-axis and x-axis. It is not a function

Example 6: Is this equation symmetric around (a) the *x*-axis? (b) the *y*-axis? (c) the origin?

$$y = x^2 + 8$$

Check for symmetry around x-axis: Replace y by −y:

$-y = x^2 + 8$

Not the same as the original, so not symmetric around x-axis

Check for symmetry around y-axis:

Replace x by −x:

$y = (-x)^2 + 8$

$y = x^2 + 8$

Same as original, so $\boxed{\text{symmetric around the y-axis}}$

Check for symmetry around the origin:

Replace x by −x, y by −y.

$-y = (-x)^2 + 8$

$-y = x^2 + 8$

Not same as original, so not symmetric around origin.

Example 7: Is this equation symmetric around (a) the *x*-axis? (b) the *y*-axis? (c) the origin?

$$x = y^2 + 8$$

Example 8: Is this equation symmetric around (a) the *x*-axis? (b) the *y*-axis? (c) the origin?

$$x^2 = y^2 + 8$$

Even and odd functions:

A function f is an <u>even function</u> if $f(-x) = f(x)$ for every x in the domain of f. The graph of an even function is symmetric with respect to the y-axis.

A function f is an <u>odd function</u> if $f(-x) = -f(x)$ for every x in the domain of f. The graph of an even function is symmetric with respect to the origin.

Example 9: Is the function $f(x) = 2 + x^2 + 3x^4$ even, odd, or neither?

Is it an even function?

Replace x by $-x$. (checking for y-axis symmetry)

$f(-x) = 2 + (-x)^2 + 3(-x)^4$

$= 2 + x^2 + 3x^4 = f(x)$

Same as f(x), so it is $\boxed{\text{an even function}}$

Is it an odd function?

Replace x by $-x$. (check for origin symmetry)

$f(-x) = 2 + (-x)^2 + 3(-x)^4$

$f(-x) = 2 + x^2 + 3x^4$

$-f(x) = -(2 + x^2 + 3x^4)$

$= -2 - x^2 - 3x^4$

not the same, so not odd

Example 10: Is the function $f(x) = x^3 + x^5$ even, odd, or neither?

Is it an even function?

Replace x by $-x$:

$f(-x) = (-x)^3 + (-x)^5$

$= -x^3 - x^5$

$\neq f(x)$, so $\boxed{\text{not even}}$

Is it an odd function?

Compare $f(-x)$ with $-f(x)$.

$f(-x) = (-x)^3 + (-x)^5$

$= -x^3 - x^5$

$-f(x) = -(x^3 + x^5)$

$= -x^3 - x^5$

f(-x) and -f(x) are equal, so $\boxed{\text{this is an odd function.}}$

Example 11: Is the function $f(x) = x^3 + 1$ even, odd, or neither?

Is it an even function?

$f(-x) = (-x)^3 + 1$

$= -x^3 + 1 \neq f(x)$

so f is $\boxed{\text{not even}}$

Is it an odd function?

$f(-x) = (-x)^3 + 1$

$= -x^3 + 1$

$-f(x) = -(x^3 + 1)$

$= -x^3 - 1$

$f(-x) \neq -f(x)$, so f is not odd.

$\boxed{\text{f is neither odd nor even.}}$

Example 12: State whether each is the graph of a function that is odd, even or neither.