1314-2-5-Notes-transformations

Thursday, September 26, 2019 10:52 AM

1314-2-5-Notes-transformations

2.5: Transformations of Functions

Learn the graphs of these frequently encountered functions.

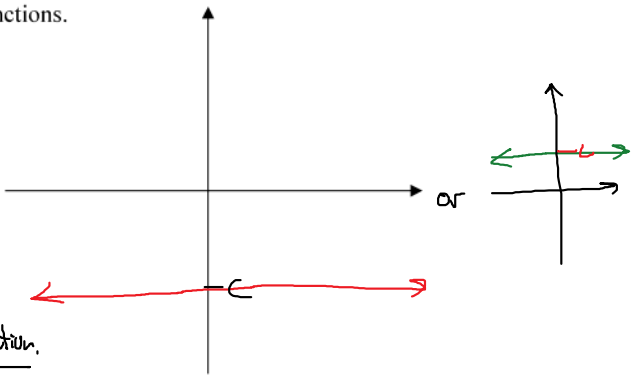
Constant function $f(x) = c$

Domain: $(-\infty, \infty)$

Range: $\{c\}$

Is it odd or even? or neither?

Symmetric around y-axis, so it is an even function.



Identity function $f(x) = x$

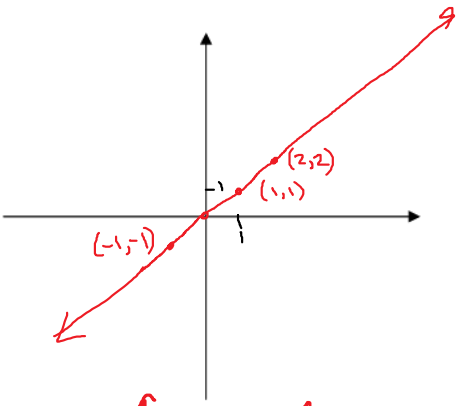
Rewrite: $y = x$, $y = 1x + 0$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Is it odd or even? or neither?

Symmetric around origin, so it is an odd function



Standard quadratic function $f(x) = x^2$

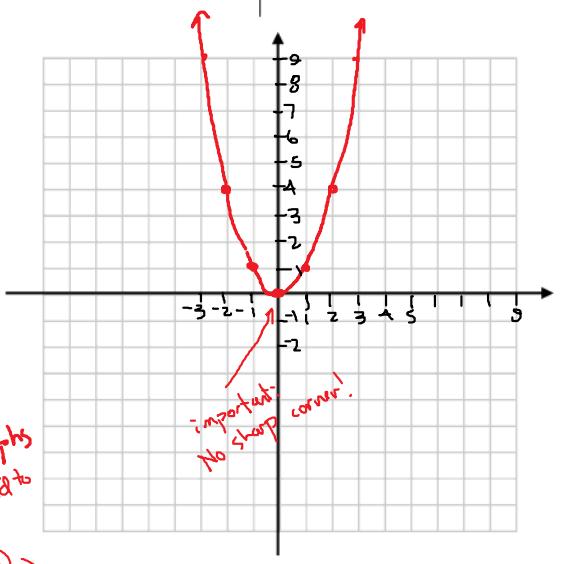
Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Is it odd or even? or neither? even function

x	$f(x) = x^2$	
-3	$f(-3) = (-3)^2 = 9$	$\Rightarrow (-3, 9)$
-2	$f(-2) = (-2)^2 = 4$	$\Rightarrow (-2, 4)$
-1	$f(-1) = (-1)^2 = 1$	
0	$f(0) = (0)^2 = 0$	
1	$f(1) = (1)^2 = 1$	
2	$f(2) = (2)^2 = 4$	$\Rightarrow (2, 4)$
3	$f(3) = (3)^2 = 9$	

all parabolas have (0,0) (±1,1) (±2,4)



Standard cubic function $f(x) = x^3$

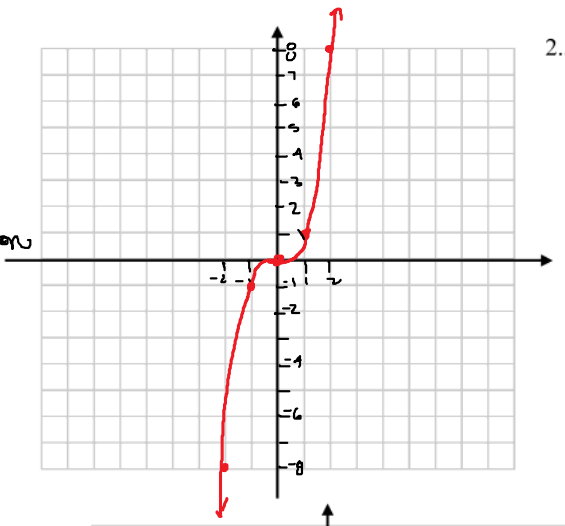
Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Is it odd or even? or neither? odd function

x	$f(x) = x^3$
-2	$f(-2) = (-2)^3 = -8$
-1	$f(-1) = (-1)^3 = -1$
0	$f(0) = (0)^3 = 0$
1	$f(1) = (1)^3 = 1$
2	$f(2) = (2)^3 = 8$

Know these 5 points!



Square root function $f(x) = \sqrt{x}$:

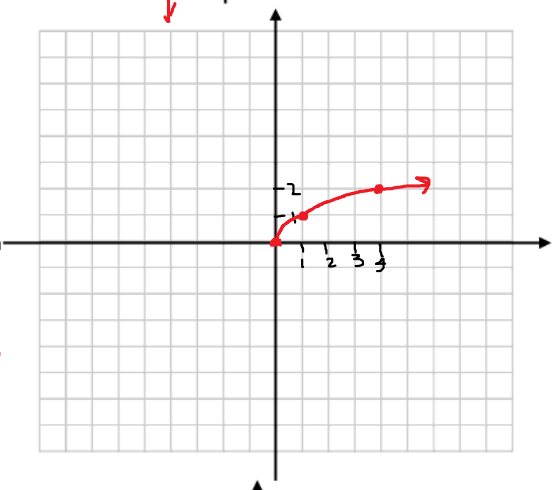
Domain: $[0, \infty)$

Range: $[0, \infty)$

Is it odd or even? or neither? neither odd nor even

x	$f(x) = \sqrt{x}$
0	$f(0) = \sqrt{0} = 0 \Rightarrow (0,0)$
1	$f(1) = \sqrt{1} = 1 \Rightarrow (1,1)$
4	$f(4) = \sqrt{4} = 2 \Rightarrow (4,2)$

Know these points



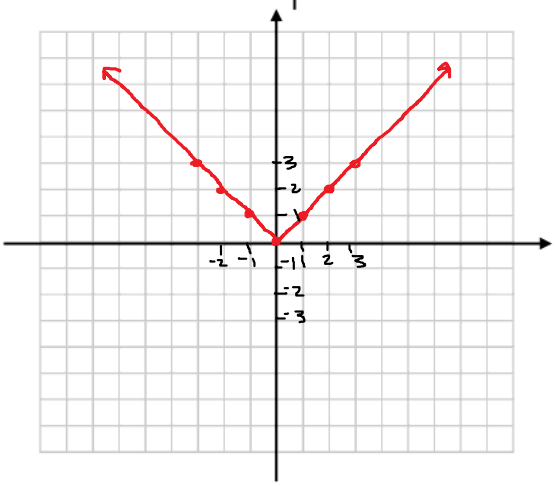
Absolute value function $f(x) = |x|$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Is it odd or even? or neither? even function

x	$f(x) = \lvert x \rvert$
-3	$f(-3) = \lvert -3 \rvert = 3 \Rightarrow (-3,3)$
-2	$f(-2) = \lvert -2 \rvert = 2 \Rightarrow (-2,2)$
-1	$f(-1) = \lvert -1 \rvert = 1 \Rightarrow (-1,1)$
0	$f(0) = \lvert 0 \rvert = 0 \Rightarrow (0,0)$
1	$f(1) = \lvert 1 \rvert = 1 \Rightarrow (1,1)$
2	$f(2) = \lvert 2 \rvert = 2 \Rightarrow (2,2)$
3	$f(3) = \lvert 3 \rvert = 3 \Rightarrow (3,3)$



If the graph of a function is known, similar functions can be graphed by varying them in several ways.

- Vertical translation (shifting vertically)
- Horizontal translation (shifting horizontally)
- Reflecting about the x-axis
- Reflecting about the y-axis
- Vertical stretching and shrinking

Translation of functions:

To *translate* a graph means to shift it horizontally, vertically, or both.

Horizontal Translation:

- Replacing x by $x-c$, with c positive, shifts the graph c units to the right.
- Replacing x by $x+c$, with c positive, shifts the graph c units to the left.

Vertical Translation:

- Adding a positive number d to the function shifts the graph upward by d units.
- Subtracting a positive number d from the function shifts the graph downward by d units.

Note:

- Adding a positive number d to the function (upward shift) is equivalent to replacing y by $y-d$.

$$y = f(x) + d$$
$$y - d = f(x)$$

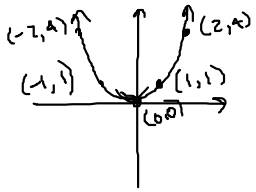
- Subtracting a positive number d (downward shift) from the function is equivalent to replacing y by $y+d$.

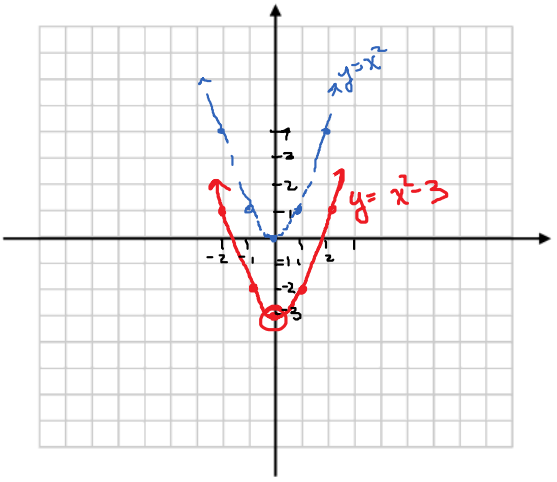
$$y = f(x) - d$$
$$y + d = f(x)$$

Example 1: Sketch the graph of $f(x) = x^2 - 3$.

This is a modification of $y = x^2$, the "parent function"

Start with $y = x^2$, then shift it down 3



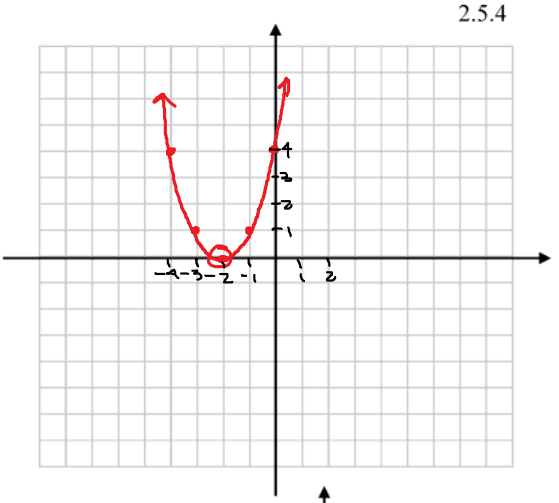


Example 2: Sketch the graph of $g(x) = (x+2)^2$.

Parent function: $y = x^2$

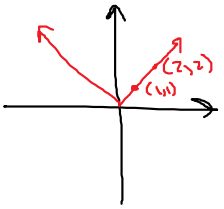
Start with $y = x^2$, then shift it left 2

Note: If $x+2=0$ then $x = -2$



Example 3: Sketch the graph of $f(x) = |x-4|$.

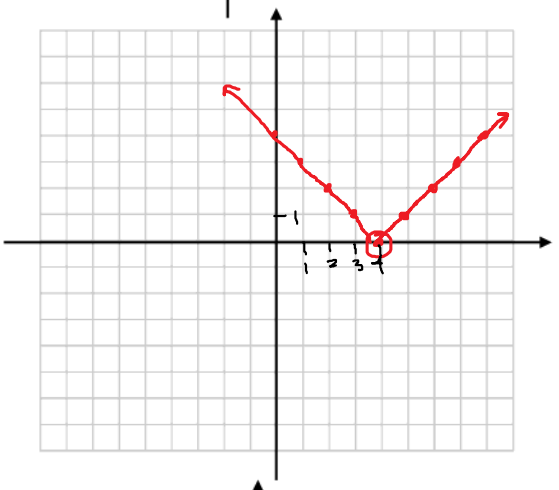
Parent function: $y = |x|$



Start with $y = |x|$, then shift it right 4

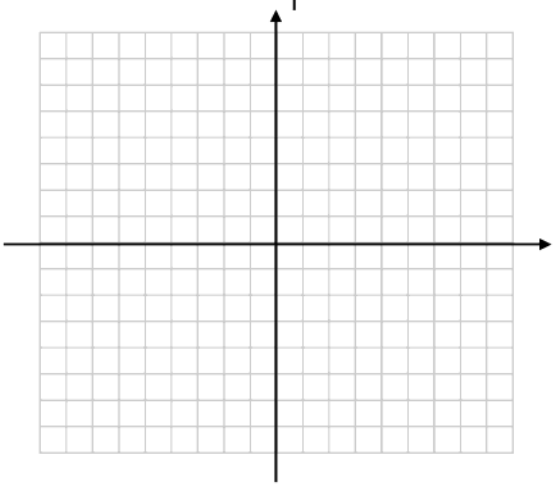
Note: $x - 4 = 0$

$x = 4 \Rightarrow$ right 4



Example 4: Sketch the graph of $f(x) = (x+2)^2 + 1$.

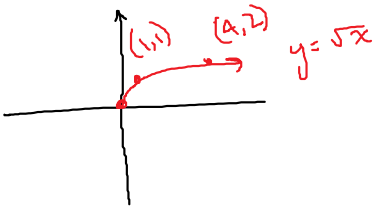
Parent

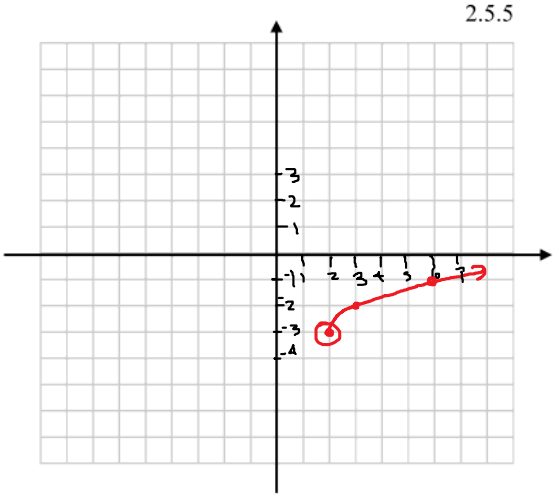


Example 5: Sketch the graph of $y = \sqrt{x-2} - 3$.

Parent function: $y = \sqrt{x}$

Start with $y = \sqrt{x}$, shift it right 2 and down 3





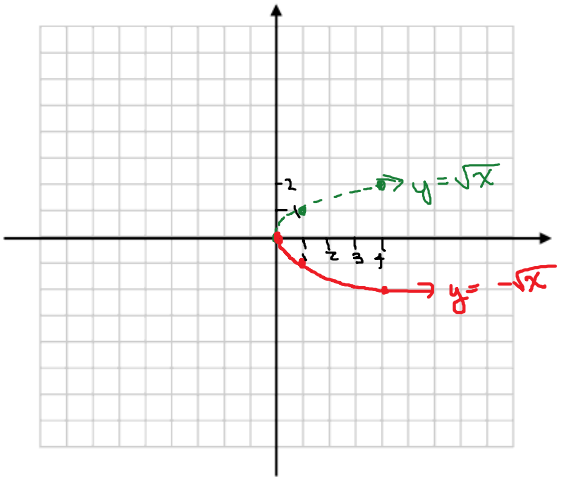
Reflection of functions:

A *reflection* is the "mirror-image" of graph about the x-axis or y-axis.

- Changing $f(x)$ to $-f(x)$ (multiplying $f(x)$ by -1 reflects the graph about the x-axis.
  (e.g. changing $y = \sqrt{x}$ to $y = -\sqrt{x}$)
- Changing $f(x)$ to $f(-x)$ (replacing x by $-x$) reflects the graph about the y-axis.
  (e.g. changing $y = \sqrt{x}$ to $y = \sqrt{-x}$)

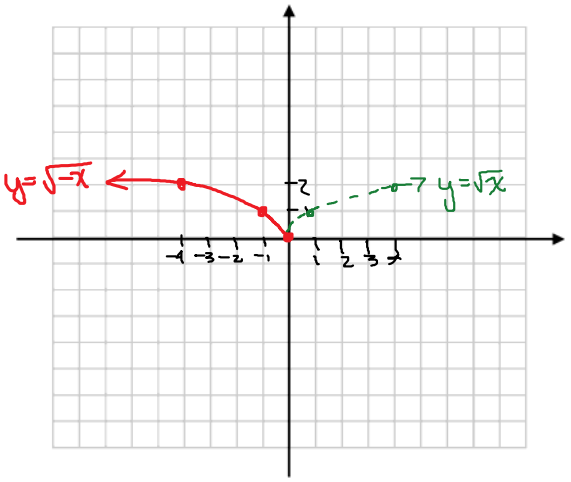
Example 6: Sketch the graph of $f(x) = -\sqrt{x}$.

Parent function $y = \sqrt{x}$, then reflect around x-axis



Example 7: Sketch the graph of $f(x) = \sqrt{-x}$.

Start with $y = \sqrt{x}$, then reflect it around y-axis



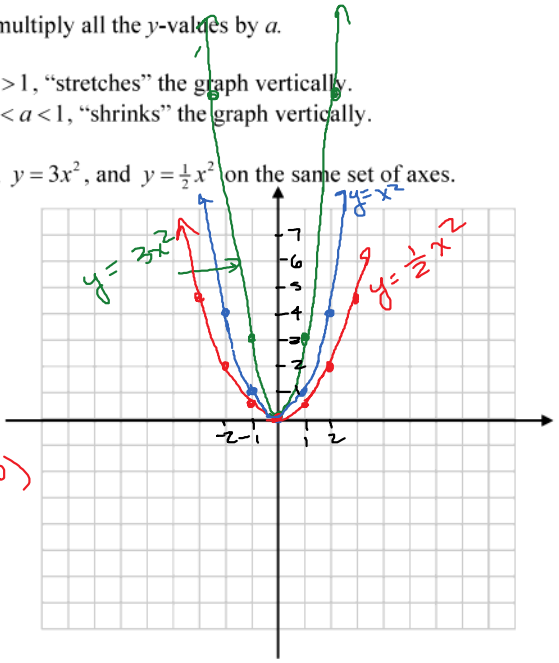
Vertical stretching and shrinking:

Multiplying a function by a number a will multiply all the y-values by a.

- Multiplying a function by a, with $a > 1$, "stretches" the graph vertically.
- Multiplying a function by a, with $0 < a < 1$, "shrinks" the graph vertically.

Example 8: Sketch the graphs of $y = x^2$, $y = 3x^2$, and $y = \frac{1}{2}x^2$ on the same set of axes.

x	$y = 3x^2$
0	$3(0)^2 = 0$
± 1	$3(\pm 1)^2 = 3$
± 2	$3(\pm 2)^2 = 12$



for all, Range is $[0, \infty)$
Domain is $(-\infty, \infty)$

Combinations of transformations:

Recommended order for transformations:

1. Stretching and shrinking
2. Reflection about x-axis.
3. Translations (horizontal and vertical).
4. Reflection about y-axis. (important to do this last!)

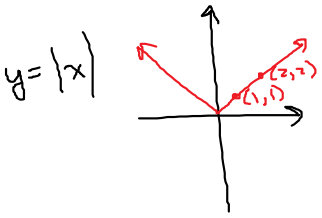
This is not the only order that works, but it is safest, and avoids having to perform algebraic manipulation of negative signs, etc.

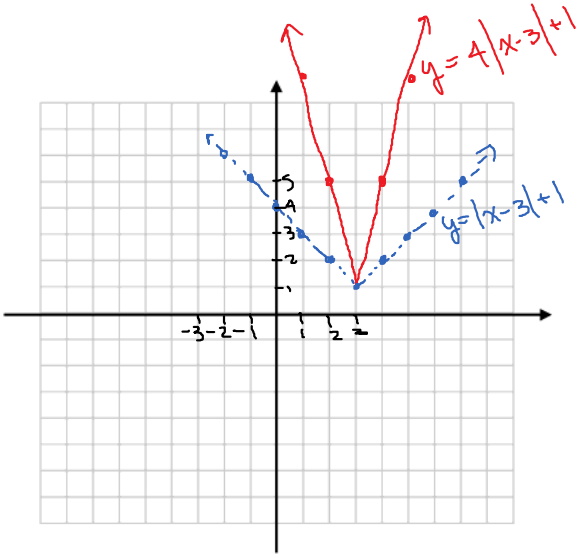
Example 9: Sketch the graph of $f(x) = 4|x-3|+1$.

Parent function: $y = |x|$

Multiply all y-values by 4.

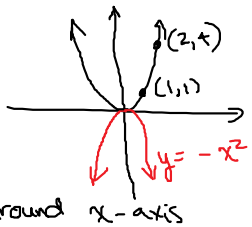
Then shift right 3, up 1





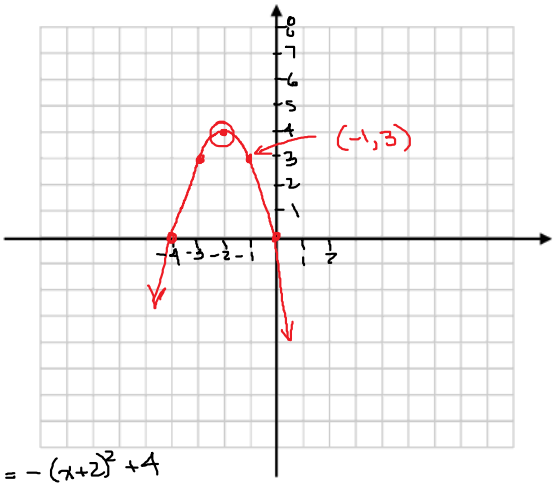
Example 10: Sketch the graph of $f(x) = -(x+2)^2 + 4$.

"Parent" function: $y = x^2$



Reflect around x-axis

Shift it left 2, up 4



Spot-check: Is $(-1,3)$ on graph of $y = -(x+2)^2 + 4$

$x = -1, y = 3 \Rightarrow$

$y = -(x+2)^2 + 4$

$3 = -(-1+2)^2 + 4$

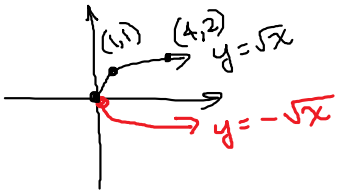
$3 = -(1)^2 + 4$

$3 = -1 + 4$

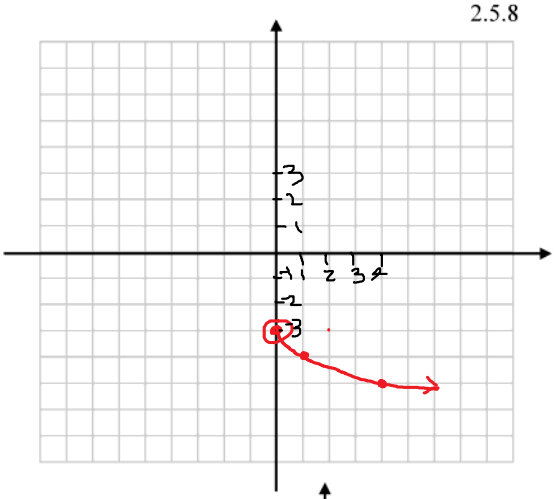
$3 = 3$ ✓

Example 11: Sketch the graph of $y = -\sqrt{x} - 3$.

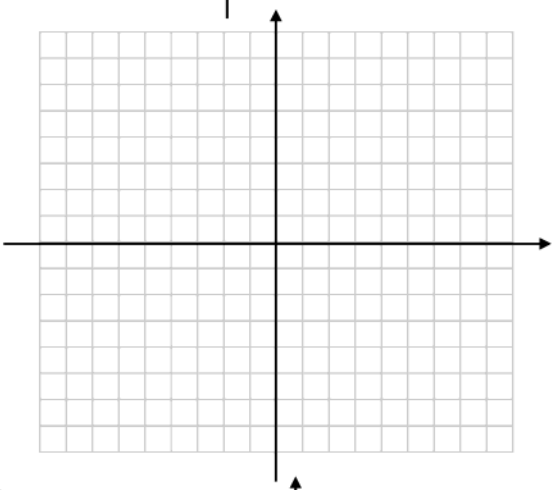
Parent function: $y = \sqrt{x}$



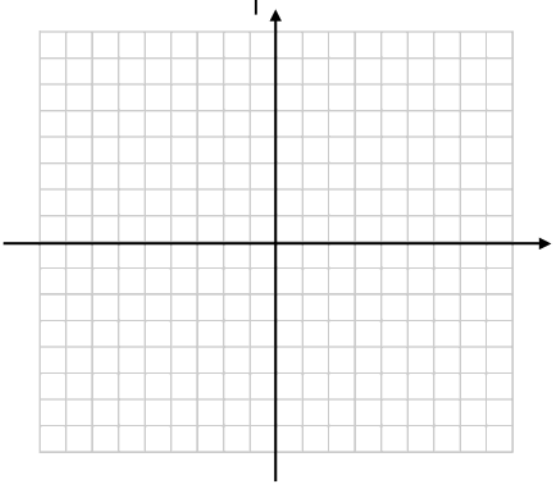
Reflect it around x-axis
Shift it down 3



Example 12: Sketch the graph of $y = \frac{1}{2}x^3 - 2$.



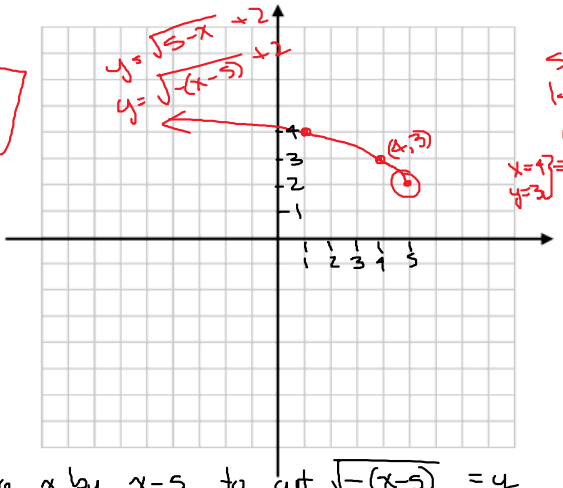
Example 13: Sketch the graph of $g(x) = -(x+1)^3 - 2$.



Example 14: Sketch the graph of $y=\sqrt{5-x}+2$.

Rewrite: $y=\sqrt{-x+5}+2$

$y=\sqrt{-(x-5)}+2$



Spot-check: Is (4,3) on graph?

$y=\sqrt{5-x}+2$

$x=4 \Rightarrow 3=\sqrt{5-4}+2$

$y=3$

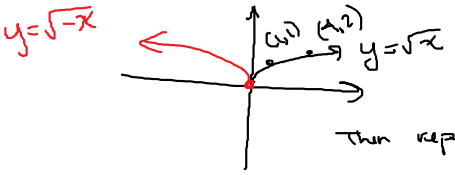
$3=\sqrt{1}+2$

$3=1+2$

$3=3$ ✓

Parent function: $y=\sqrt{x}$

To graph $y=\sqrt{-x}$, reflect around y-axis



Then replace x by x−5 to get $\sqrt{-(x-5)}=y$ (this shifts it right 5). Then add 2 to shift up 2.

Example 15: Given the graph of $f(x)$, sketch the graph of

a. $-f(x)$

b. $f(-x)$

c $f(x-2)$