1314-3-1-Notes-quadratic-fcns

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3.1: Quadratic Functions

A quadratic function is a function which can be written in the form $f(x) = ax^2 + bx + c$ ($a \neq 0$).

Its graph is a parabola.

Definition: The *maximum* value of a function is the largest y-value on the graph. The *minimum* value of a function is the smallest y-value on the graph.

Standard form for a quadratic function:

Every quadratic function $f(x) = ax^2 + bx + c$ can be written in the form

$$f(x) = a(x-h)^2 + k\,.$$

This is called *standard form* for a quadratic function.

- The vertex of the parabola is (h, k).
- If $a < 0$, the graph opens down and has a *maximum* value.
- If $a > 0$, the graph opens up and has a *minimum* value.
- The larger $|a|$, is the narrower the parabola is.

To write a quadratic function $f(x) = ax^2 + bx + c$ in standard form, we need to complete the square.

Example 1: Sketch the graph of $f(x) = \cancel{3}\,2x^2 + 16x - 35$. State the vertex and intercepts. What is the maximum or minimum value?

$f(x) = 2x^2 + 16x - 35$

$y = (2x^2 + 16x) - 35$

$y = 2(x^2 + 8x) - 35$

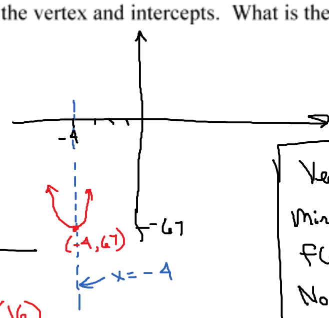
$y = 2(x^2 + 8x + \_\_\_) - 35 + \_\_\_$

$\left(\frac{8}{2}\right)^2 = (4)^2 = 16$

$y = 2(x^2 + 8x + \underline{16}) - 35 + \underline{-2(16)}$

$y = 2(x+4)^2 - 35 - 32$

Standard form: $\boxed{f(x) = 2(x+4)^2 - 67}$



Start with $y = x^2$, multiply y-values by 2 (stretch), shift it left 4, down 67

Vertex: $(-4, -67)$

Minimum value:

$f(-4) = -67$

No maximum value

Axis of symmetry: $x = -4$

Find y-intercept: Set $x=0$

$y = 2x^2 + 16x - 35$

$x=0$: $y = 2(0)^2 + 16(0) - 35$

$y = -35 \Rightarrow$ $\boxed{y\text{-intercept is } (0, -35)}$

or, $\boxed{y\text{-intercept: } -35}$

Ex 1 (cont'd.): Find x-intercepts: Set $y=0$.

$y = 2(x+4)^2 - 67$

$0 = 2(x+4)^2 - 67$

$\boxed{(0,-35)}$

or, $\boxed{\text{y-intercept: } -35}$

Ex 1 contd: Find x-intercepts: Set $y=0$.

$$y = 2(x+4)^2 - 67$$
$$0 = 2(x+4)^2 - 67$$
$$67 = 2(x+4)^2$$
$$\frac{67}{2} = (x+4)^2$$
$$\sqrt{(x+4)^2} = \pm\sqrt{\frac{67}{2}}$$

$$x + 4 = \pm\sqrt{\frac{67}{2}}$$
$$x + 4 = \pm\frac{\sqrt{67}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$
$$x + 4 = \pm\frac{\sqrt{134}}{2}$$
$$x = -4 \pm \frac{\sqrt{134}}{2}$$

$\boxed{\text{x-intercepts: } -4 \pm \frac{\sqrt{134}}{2}}$

3.1.2

Example 2: Sketch the graph of $f(x) = -3x^2 + 15x - 18$. State the vertex and intercepts. What is the maximum or minimum value?

$$y = -3x^2 + 15x - 18$$
$$y = (-3x^2 + 15x) - 18$$
$$y = -3(x^2 - 5x) - 18$$
$$y = -3\left(x^2 - 5x + \frac{25}{4}\right) - 18 + 3\left(\frac{25}{4}\right)$$
$$\left(\frac{-5}{2}\right)^2 = \frac{25}{4}$$
$$y = -3\left(x - \frac{5}{2}\right)^2 - \frac{18}{1}\cdot\left(\frac{4}{4}\right) + \frac{75}{4}$$
$$y = -3\left(x - \frac{5}{2}\right)^2 - \frac{72}{4} + \frac{75}{4}$$

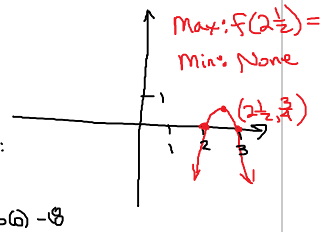
Std form: $\boxed{f(x) = -3\left(x - \frac{5}{2}\right)^2 + \frac{3}{4}}$

Start with $y = x^2$, stretch by a factor of 3, reflect around x-axis, shift it right by $\frac{5}{2} = 2\frac{1}{2}$, shift it up $\frac{3}{4}$.

$\boxed{\text{Vertex: } \left(2\tfrac{1}{2}, \tfrac{3}{4}\right)}$

Max: $f\left(2\frac{1}{2}\right) = \frac{3}{4}$

Min: None

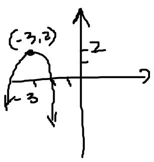


Find y-intercept:
Set $x=0$:
$$y = -3(0)^2 + 15(0) - 18$$
$$y = -18 \Rightarrow \boxed{(0,-18)}$$

Find x-intercepts: Set $y=0$
$$0 = -3x^2 + 15x - 18$$
$$0 = -3(x^2 - 5x + 6)$$
$$0 = -3(x-2)(x-3)$$
$$x = 2, 3$$

$\boxed{\text{x-intercepts: } (2,0) \text{ and } (3,0)}$

Example 3: Sketch the graph of $f(x) = -2x^2 - 12x - 16$.



maximum is $f(-3) = 2$
No minimum

$$y = -2(x+3)^2 + 2$$

Vertex: $(-3, 2)$ Opens down

Example 4: Write $f(x) = x^2 + x + 2$ in standard form. What is its maximum or minimum value? What input produces this maximum or minimum?

$$y = x^2 + x + 2$$
$$y = (x^2 + x) + 2$$
$$y = \left(x^2 + 1x + \frac{1}{4}\right) + 2 + -\frac{1}{4}$$
$$\left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2} = \frac{1}{4}$$
$$y = \left(x + \frac{1}{2}\right)^2 + \frac{2}{1}\cdot\left(\frac{4}{4}\right) - \frac{1}{4}$$
$$y = \left(x + \frac{1}{2}\right)^2 + \frac{8}{4} - \frac{1}{4}$$

Opens up, so it has a minimum.

Vertex: $\left(-\frac{1}{2}, \frac{7}{4}\right)$

Minimum: $f\left(-\frac{1}{2}\right) = \frac{7}{4}$

or $\boxed{f\left(-\tfrac{1}{2}\right) = 1\tfrac{3}{4}}$

Std. Form

$\Rightarrow \boxed{f(x) = \left(x + \frac{1}{2}\right)^2 + \frac{7}{4}}$

Example 5: Find the maximum or minimum value of $f(x) = -2x^2 - 5x - 1$.

$y = -2x^2 - 5x - 1$

$y = (-2x^2 - 5x) - 1$

$y = -2\left(x^2 - \frac{5}{-2}x\right) - 1$

$y = -2\left(x^2 + \frac{5}{2}x\right) - 1$

$y = -2\left(x^2 + \frac{5}{2}x + \frac{25}{16}\right) - 1 + \frac{2}{1}\left(\frac{25}{16}\right)$

$\left(\frac{5/2}{2}\right)^2 = \left(\frac{5}{2} \cdot \frac{1}{2}\right)^2 = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$

$y = -2\left(x^2 + \frac{5}{2}x + \frac{25}{16}\right) - 1 + \frac{50}{16}$

$y = -2\left(x + \frac{5}{4}\right)^2 - \frac{16}{16} + \frac{50}{16}$

$\boxed{f(x) = -2\left(x + \frac{5}{4}\right)^2 + \frac{34}{16}}$

Example 6: Find the quadratic function such that $f(3) = -6$ and the vertex is $(-2, -3)$.