1314-3-3-Notes-poly-division

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3.3: Dividing Polynomials; Remainder and Factor Theorems

Terms you should know:

$$\text{Divisor}\overline{\big)\text{Dividend}}^{\ \text{Quotient}}$$

*****
*****
*****
Remainder

Recall: Dividend = (Divisor)(Quotient) + Remainder

So… $\dfrac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \dfrac{\text{Remainder}}{\text{Divisor}}$

Example 1: Divide $\dfrac{379}{12}$.

$$12\overline{)379}^{\ 31}$$
-36
19
12
7

So, $\frac{379}{12} = 379 \div 12$

$= \boxed{31 + \frac{7}{12}}$

$= 31\frac{7}{12}$

Check
$12(31) + 7 = 372 + 7 = 379$

Long division of polynomials will be done similarly…

Example 2: Divide $\dfrac{3x^2+7x+9}{x+2}$.

$\frac{3x^2}{x}$ $\frac{x}{x}$

$$x+2\overline{)3x^2 + 7x + 9}^{\ 3x + 1}$$
$-(3x^2 + 6x)$
$x + 9$
$-(x + 2)$
7

$\dfrac{3x^2+7x+9}{x+2} = \boxed{3x+1 + \dfrac{7}{x+2}}$

Check:
$(x+2)(3x+1) + 7$
$= 3x^2 + 7x + 9$

Recall:
$\dfrac{4x^2+8x+9}{2x}$
$= \dfrac{4x^2}{2x} + \dfrac{8x}{2x} + \dfrac{9}{2x}$
$= 2x + 4 + \dfrac{9}{2x}$

Important:

You *must* put zeros in place of any missing powers of x.
The 0's act as "placeholders" just as they do in our number system.

For example, 206 is certainly not the same number as 26. The 0 "holds" the 10's place.

Similarly, when doing division, we write $2x^2+0x+6$ instead of $2x^2+6$.

Example 3: Divide $\frac{8x^4-10x^2+3x-2}{2x-1}$.

$$\frac{8x^4-10x^2+3x-2}{2x-1}$$

$$= \boxed{4x^3+2x^2-4x-\frac{1}{2}+\frac{-\frac{5}{2}}{2x-1}}$$

$$= \boxed{4x^3+2x^2-4x-\frac{1}{2}-\frac{\frac{5}{2}}{2x-1}}$$

$\frac{8x^4}{2x}$ $\frac{4x^3}{2x}$ $\frac{-8x^2}{2x}$ $\frac{-x}{2x}=\frac{-1x}{2x}$

$$
\begin{array}{r}
4x^3 + 2x^2 - 4x - \frac{1}{2} \\
2x-1 \overline{) \; 8x^4 + 0x^3 - 10x^2 + 3x - 2} \\
+ \; -(8x^4 - 4x^3) \\
\hline
4x^3 - 10x^2 \\
+ \; -(4x^3 - 2x^2) \\
\hline
-8x^2 + 3x \\
+ \; -(-8x^2 + 4x) \\
\hline
-x - 2 \\
+ \; -(-x + \frac{1}{2}) \\
\hline
-\frac{5}{2}
\end{array}
$$

$-\frac{1}{2}(2x-1) = -x+\frac{1}{2}$

$-2-\frac{1}{2} = -\frac{4}{2}-\frac{1}{2} = -\frac{5}{2}$

Example 4: Divide $\frac{x^4-5x^3+6x-8}{x^2+2}$.

Example 5: Divide $\dfrac{x^3-2x^2-5x+6}{x-3}$.

You can use this long division to factor x^3-2x^2-5x+6…

The Factor Theorem

Let $f(x)$ be a polynomial,

- If $f(c)=0$, then $x-c$ is a factor of $f(x)$.
- If $x-c$ is a factor of $f(x)$, then $f(c)=0$. (c is a zero of f)

In other words, c is a zero (root, x-intercept) of $f(x)$ if and only if $x-c$ is a factor of $f(x)$.

Example 6: Show that 4 is a zero of the function $P(x)=x^3+3x^2-18x-40$. Use this fact to factor P completely and find all other zeros of $P(x)$.

$$P(x)=x^3+3x^2-18x-40$$
$$P(4)=(4)^3+3(4)^2-18(4)-40$$
$$=64+3(16)-72-40$$
$$=64+48-72-40$$
$$=112-112$$
$$=0$$

So 4 is a zero of $P(x)$.
The $x-4$ is a factor of $P(x)$.

$$\begin{array}{r|rrrr} 4 & 1 & 3 & -18 & -40 \\ & & 4 & 28 & 40 \\ \hline & 1 & 7 & 10 & \boxed{0} \end{array}$$

Remainder.

$$P(x)=x^3+3x^2-18x-40$$
$$=(x-4)(x^2+7x+10)$$
$$\boxed{P(x)=(x-4)(x+2)(x+5)}$$

Zeros are $4, -2, -5$

Synthetic Division: This is a "shortcut" which can be used to divide a polynomial by $x-c$, where c is a real number. Synthetic division does *not* work for divisors like x^2+1, x^2+2x-7, etc.

$\frac{3x^3}{x}$ $\frac{10x^2}{x}$

Example 7: Divide $\dfrac{3x^3+4x^2-13}{x-2}$.

Example 7: Divide $\frac{3x^3+4x^2-13}{x-2}$.

Do it with long division first:

$\frac{3x^3}{x}$ $\quad$ $\frac{10x^2}{x}$

$$
\begin{array}{r|l}
 & 3x^2 + 10x + 20 \\
\hline
x-2 & 3x^3 + 4x^2 + 0x - 13 \\
 & -(3x^3 - 6x^2) \\
\hline
 & 10x^2 + 0x \\
 & -(10x^2 - 20x) \\
\hline
 & 20x - 13 \\
 & -(20x - 40) \\
\hline
 & 27
\end{array}
$$

$$\frac{3x^3+4x^2-13}{x-2} = \boxed{3x^2+10x+20+\frac{27}{x-2}}$$

Check: $3x^3+4x^2-13 = (x-2)(3x^2+10x+20)+27$

With synthetic division, the above work can be reduced to:

2	3	4	0	−13
		6	20	40
	3	10	20	27

Notice: If $x-2$ is a factor, then 2 is a zero (x-intercept)

The divisor $x-2$ is associated with a zero of 2

2	3	4	0	−13
		6	20	40
	3	10	20	27

$\underbrace{3 \quad 10 \quad 20}_{\text{coefficients of quotient}}$

or

2	3	4	0	−13

Degree of quotient is always 1 less than the degree of the dividend.

Dividend $3x^3+4x^2-13$ is degree 3, so quotient is degree 2.

quotient is $3x^2+10x+20$ and so $\frac{3x^3+4x^2-13}{x-2} = \boxed{3x^2+10x+20+\frac{27}{x-2}}$

How to do synthetic division:

Step 1: Write the dividend in descending powers. Then, copy the coefficients and insert 0 for any missing powers.

$$\begin{array}{rrrr} 3 & 4 & 0 & -13 \end{array}$$

Step 2: Write the usual division symbol over the numbers. If the divisor is $x-c$, write "c" to the left. Leave space under the numbers and draw a horizontal line.

$$\begin{array}{r|rrrr} 2 & 3 & 4 & 0 & -13 \\ & & & & \\ \hline & & & & \end{array}$$

Step 3: Bring down the first number under the division sign.

$$\begin{array}{r|rrrr} 2 & 3 & 4 & 0 & -13 \\ & & & & \\ \hline & 3 & & & \end{array}$$

Step 4: Multiply the latest entry of Row 3 by the number "outside" and write that product on Row 2 in the next column.

$$\begin{array}{r|rrrr} 2 & 3 & 4 & 0 & -13 \\ & & 6 & & \\ \hline & 3 & & & \end{array}$$

Step 5: Add the numbers in the new column and write the sum on Row 3.

$$\begin{array}{r|rrrr} 2 & 3 & 4 & 0 & -13 \\ & & 6 & & \\ \hline & 3 & 10 & & \end{array}$$

Step 6: Repeat Step 4 and Step 5 until all the columns have been used.

$$\begin{array}{r|rrrr} 2 & 3 & 4 & 0 & -13 \\ & & 6 & 20 & \\ \hline & 3 & 10 & 20 & \end{array} \qquad \begin{array}{r|rrrr} 2 & 3 & 4 & 0 & -13 \\ & & 6 & 20 & 40 \\ \hline & 3 & 10 & 20 & 27 \end{array}$$

Step 7: The rightmost number on Row 3 is the remainder. The remaining entries on Row 3 are the coefficients of the quotient. The degree of the quotient is one less than the degree of the dividend.

Remainder: 27

Quotient: $3x^2+10x+20$

Therefore,

$$3x^3+4x^2-13=(x-2)(3x^2+10x+20)+\frac{27}{x-2}$$

Example 8: Divide $\dfrac{x^3-2x^2-5x+6}{x-3}$.

Example 9: Divide $\dfrac{x^4+7x^3+8x^2-28x-40}{x+3}$.

$x+3=0$
$x=-3$

$$\begin{array}{r|rrrrr} -3 & 1 & 7 & 8 & -28 & -40 \\ & & -3 & -12 & 12 & 48 \\ \hline & 1 & 4 & -4 & -16 & \boxed{8} \end{array}$$

$$\frac{x^4+7x^3+8x^2-28x-40}{x+3}$$

$$= \boxed{x^3+4x^2-4x-16+\frac{8}{x+3}}$$

<u>Check:</u>

$$x^4+7x^3+8x^2-28x-40 = (x+3)(x^3+4x^2-4x-16)+8$$

Example 10: Divide $\dfrac{x^3+3x^2-18x-40}{x-4}$.

Example 11: Show that 3 is a zero of the function $P(x) = 2x^3 + 7x^2 - 19x - 60$. Use this fact to factor P completely and find all other zeros of $P(x)$.

For 3 to be a zero, we should get a remainder of 0 when we divide $P(x)$ by $x-3$.

3	2	7	-19	-60
		6	39	60
	2	13	20	$\boxed{0}$

Remainder is 0, so 3 is a zero.

$P(x) = 2x^3 + 7x^2 - 19x - 60$

$= (x-3)(2x^2 + 13x + 20)$

$\boxed{P(x) = (x-3)(2x+5)(x+4)}$

$x-3=0$, $x=3$ | $2x+5=0$, $2x=-5$, $x=-\frac{5}{2}$ | $x+4=0$, $x=-4$

$220 = 40$

1·40
2·20
4·10
5·8

Remainder Theorem:

If the polynomial $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.

$\boxed{\text{Zeros are } 3, -\frac{5}{2}, -4.}$

Example 12: Given $f(x) = x^4 + 9x^3 + 5x^2 - 12x - 13$, use the Remainder Theorem to find $f(-4)$.

So divisor is $x+4$.

-4	1	9	5	-12	-13
		-4	-20	60	-192
	1	5	-15	48	$\boxed{-205}$

$48 \times 4 = 192$

So $\boxed{f(-4) = -205}$

Example 13: Solve the equation $x^4 + 7x^3 + 8x^2 - 28x - 48 = 0$ given that -3 is a zero of $P(x) = x^4 + 7x^3 + 8x^2 - 28x - 48$.

-3	1	7	8	-28	-48
		-3	-12	12	48
	1	4	-4	-16	$\boxed{0}$

$P(x) = x^4 + 7x^3 + 8x^2 - 28x - 48$

$= (x+3)(x^3 + 4x^2 - 4x - 16)$

$= (x+3)[(x^3 + 4x^2) + (-4x - 16)]$

$= (x+3)[x^2(x+4) - 4(x+4)]$

$= (x+3)[(x+4)(x^2 - 4)]$

$= (x+3)(x+4)(x+2)(x-2) = 0$

Then $x = -3, -4, -2, 2$

Solution Set:

$\boxed{\{-3, -4, \pm 2\}}$

Example: Divide $\dfrac{6x^3-19x^2-65x+50}{2x+5}$.

Step 1: Divide $\dfrac{6x^3}{2x}=$

$$\begin{array}{r} 3x^2 \\ 2x+5\overline{)6x^3-19x^2-65x+50} \end{array}$$

Step 2: Multiply $3x^2$ by $2x+5$

$$\begin{array}{r} 3x^2 \qquad\qquad\quad \\ 2x+5\overline{)6x^3-19x^2-65x+50} \\ \underline{6x^3+15x^2} \qquad\qquad \end{array}$$

Step 3: Subtract (Don't forget the parentheses!)

$$\begin{array}{r} 3x^2 \qquad\qquad\quad \\ 2x+5\overline{)6x^3-19x^2-65x+50} \\ -\underline{(6x^3+15x^2)} \qquad\qquad \\ -34x^2 \qquad\qquad \end{array}$$

Step 4: Bring Down

$$\begin{array}{r} 3x^2 \qquad\qquad\quad \\ 2x+5\overline{)6x^3-19x^2-65x+50} \\ -\underline{(6x^3+15x^2)} \qquad\qquad \\ -34x^2-65x \qquad \end{array}$$

Repeat the steps…

Step 1: Divide $\dfrac{-34x^2}{2x}=$

$$\begin{array}{r} 3x^2-17x \qquad \\ 2x+5\overline{)6x^3-19x^2-65x+50} \\ -\underline{(6x^3+15x^2)} \qquad\qquad \\ -34x^2-65x \qquad \end{array}$$

Step 2: Multiply $-17x$ by $2x+5$

$$\begin{array}{r} 3x^2-17x \qquad \\ 2x+5\overline{)6x^3-19x^2-65x+50} \\ -\underline{(6x^3+15x^2)} \qquad\qquad \\ -34x^2-65x \qquad \\ -34x^2-85x \qquad \end{array}$$

Go to the top right…

Step 4: Bring Down

$$\begin{array}{r} 3x^2-17x \qquad \\ 2x+5\overline{)6x^3-19x^2-65x+50} \\ -\underline{(6x^3+15x^2)} \qquad\qquad \\ -34x^2-65x \qquad \\ -\underline{(-34x^2-85x)} \qquad \\ 20x+50 \end{array}$$

Repeat the steps…

Step 1: Divide $\dfrac{20x}{2x}=$

$$\begin{array}{r} 3x^2-17x+10 \\ 2x+5\overline{)6x^3-19x^2-65x+50} \\ -\underline{(6x^3+15x^2)} \qquad\qquad \\ -34x^2-65x \qquad \\ -\underline{(-34x^2-85x)} \qquad \\ 20x+50 \end{array}$$

Step 2: Multiply 10 by $2x+5$

$$\begin{array}{r} 3x^2-17x+10 \\ 2x+5\overline{)6x^3-19x^2-65x+50} \\ -\underline{(6x^3+15x^2)} \qquad\qquad \\ -34x^2-65x \qquad \\ -\underline{(-34x^2-85x)} \qquad \\ 20x+50 \\ 20x+50 \end{array}$$

Step 3: Subtract (Don't forget the parentheses!)

$$\begin{array}{r} 3x^2-17x+10 \\ 2x+5\overline{)6x^3-19x^2-65x+50} \\ -\underline{(6x^3+15x^2)} \qquad\qquad \\ -34x^2-65x \qquad \\ -\underline{(-34x^2-85x)} \qquad \\ 20x+50 \\ -\underline{(20x+50)} \\ \text{Remainder} \rightarrow 0 \end{array}$$

Extra Handout on Polynomial Long Division