1314-3-4-rational-zeros

Monday, November 18, 2019 12:35 PM

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3.4: Zeros of Polynomial Functions

Recall: Rational Number: Can be written as a ratio (fraction, quotient) of two integers.

Ex: $\frac{2}{3}, \frac{5}{2}, 6, -5, 0.53, 1.17, 0.\overline{33}$

(Last 3 are $\frac{53}{100}, \frac{117}{100}, \frac{1}{3}$)

The Rational Zero Theorem: (Rational Roots Theorem)

If a polynomial has integer coefficients, and if $\frac{p}{q}$ is a rational zero in reduced form, then p is a factor of the constant term, and q is a factor of the leading coefficient.

Example 1: Consider $f(x) = 6x^2 - 19x - 7$. Find the ~~possible rational zeros and t~~he actual zeros.

$f(x) = 6x^2 - 19x - 7$

(-) signs opposite, want a difference of 19x for middle term

$6 \cdot 7 = 42$

$1 \cdot 42$

$2 \cdot 21$

$3 \cdot 14$

$6 \cdot 7$

$= (2x - 7)(3x + 1)$

Find the zeros. Set $f(x)=0$

$0 = (2x-7)(3x+1)$

$2x - 7 = 0$, $2x = 7$, $x = \frac{7}{2}$

$3x+1=0$, $3x = -1$, $x = -\frac{1}{3}$

Zeros are $\frac{7}{2}, -\frac{1}{3}$

Example 2: Find the possible rational zeros of $g(x) = 4x^4 - 5x^3 + 3x^2 - 13x - 18$.

Factors of constant term 18: 1, 18, 2, 9, 3, 6

Factors of leading coefficient 4: 1, 4, 2

Possible rational zeros: $\pm\left\{\frac{1}{1}, \frac{1}{4}, \frac{1}{2}, \frac{18}{1}, \frac{18}{4}, \frac{18}{2}, \frac{2}{1}, \frac{2}{4}, \frac{2}{2}, \frac{9}{1}, \frac{9}{4}, \frac{9}{2}, \frac{3}{1}, \frac{3}{4}, \frac{3}{2}, \frac{6}{1}, \frac{6}{4}, \frac{6}{2}\right\}$

$\Longrightarrow \pm\left\{1, \frac{1}{4}, \frac{1}{2}, 18, \frac{9}{2}, 9, 2, \frac{9}{4}, 3, \frac{3}{4}, \frac{3}{2}, 6\right\}$

To find zeros of a polynomial function:

- List the possible rational zeros.
- Then use synthetic division to find one that gives a zero remainder and is therefore a zero.
- Use the result of your synthetic division to factor (partially) the polynomial.
- Repeat the process until the polynomial is completely factored.

Example 3: Find the zeros of $f(x) = 2x^3 + 5x^2 - 22x + 15$.

Factors of constant term 15: 1, 15, 3, 5

Factors of lead. coeff. 2: 1, 2

Possible rational zeros: $\pm\left\{\frac{1}{1}, \frac{1}{2}, \frac{15}{1}, \frac{15}{2}, \frac{3}{1}, \frac{3}{2}, \frac{5}{1}, \frac{5}{2}\right\}$

$$\Rightarrow \pm\left\{1, \frac{1}{2}, 15, \frac{15}{2}, 3, \frac{3}{2}, 5, \frac{5}{2}\right\}$$

$$\begin{array}{r|rrrr} 1 & 2 & 5 & -22 & 15 \\ & & 2 & 7 & -15 \\ \hline & 2 & 7 & -15 & \boxed{0} \end{array}$$

So 1 is a zero and $x-1$ is a factor.

$f(x) = 2x^3 + 5x^2 - 22x + 15$

$= (x-1)(2x^2 + 7x - 15)$

$= (x-1)(2x-3)(x+5)$

$x - 1 = 0$, $x = 1$ | $2x - 3 = 0$, $2x = 3$, $x = \frac{3}{2}$ | $x + 5 = 0$, $x = -5$

$\boxed{\text{Zeros are: } 1, \frac{3}{2}, -5}$

Example 4: Find the zeros of $f(x) = 3x^3 + 8x^2 - 7x - 12$.

$f(x) = 3x^3 + 8x^2 - 7x - 12$

Factors of constant term 12: $\pm\{1, 12, 2, 6, 3, 4\}$

Factors of leading coefficient 3: $\pm\{1, 3\}$

Possible rational zeros (candidates): $\pm\left\{\frac{1}{1}, \frac{1}{3}, \frac{12}{1}, \frac{12}{3}, \frac{6}{1}, \frac{6}{3}, \frac{3}{1}, \frac{3}{3}, \frac{4}{1}, \frac{4}{3}, \frac{2}{1}, \frac{2}{3}\right\}$

$$\Rightarrow \pm\left\{1, \frac{1}{3}, 12, 4, 6, 2, 3, \frac{4}{3}, \frac{2}{3}\right\}$$

$$\begin{array}{r|rrrr} 1 & 3 & 8 & -7 & -12 \\ & & 3 & 11 & 4 \\ \hline & 3 & 11 & 4 & \boxed{-8} \end{array}$$

No!

$$\begin{array}{r|rrrr} -1 & 3 & 8 & -7 & -12 \\ & & -3 & -5 & 12 \\ \hline & 3 & 5 & -12 & \boxed{0} \end{array}$$

-1 is a zero, so $x+1$ is a factor

$f(x) = 3x^3 + 8x^2 - 7x - 12$

$= (x+1)(3x^2 + 5x - 12)$

$\boxed{f(x) = (x+1)(3x-4)(x+3)}$ f(x) in factored form

$3x - 4 = 0$
$3x = 4$
$x = \frac{4}{3}$

$\boxed{\text{Zeros: } -1, \frac{4}{3}, -3}$

12
1·12
2·6
3·4

Note: to see if $3x^2 + 5x - 12$ factors.
$3(-12) = 36$
1·32
2·18
3·12
4·9
6·6

Example 5: Solve $x^3 + 11x^2 + 26x - 8 = 0$.

$x^3 + 11x^2 + 26x - 8 = 0$

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$x^3 + 11x^2 + 26x - 8 = 0$

Example 6: Solve $x^4 - 6x^3 + x^2 + 24x - 20 = 0$.

Example 7: Solve $x^4+x^3-4x^2-16x-24=0$.

Factors of Constant term -24: $\pm\{1, 24, 2, 12, 3, 8, 4, 6\}$

Factors of Leading Coefficient 1: ± 1

Possible rational zeros: $\pm\{1, 24, 2, 12, 3, 8, 4, 6\}$

-3	1	1	-4	-16	-24
		-3	6	6	36
	1	-2	-2	-12	$\boxed{12}$ No!

3	1	1	-4	-16	-24
		3	12	24	24
	1	4	8	8	$\boxed{0}$

-2	1	4	8	8
		-2	-4	-8
	1	2	4	$\boxed{0}$

$$x^4+x^3-4x^2-16x-24=0$$
$$(x-3)(x^3+4x^2+8x+8)=0$$
$$(x-3)(x+2)(x^2+2x+4)=0$$
$$x=\frac{-2\pm\sqrt{(2)^2-4(1)(4)}}{2(1)}$$
$$=\frac{-2\pm\sqrt{4-16}}{2}=\frac{-2\pm\sqrt{-12}}{2}$$
$$=\frac{-2\pm i\sqrt{4\cdot 3}}{2}=\frac{-2\pm 2i\sqrt{3}}{2}$$
$$=\frac{-\cancel{2}}{\cancel{2}}\pm\frac{\cancel{2}i\sqrt{3}}{\cancel{2}}$$
$$=-1\pm i\sqrt{3}$$

Solution Set: $\{3, -2, -1\pm i\sqrt{3}\}$

Important Facts:

- A polynomial of degree n has n real or complex roots, counting multiplicities. So it can be written as a product of n factors.
- If a non-real number is the root of a polynomial, then its complex conjugate is also a root.

Example 8: $2i$ is a root of the equation $x^4-9x^3+22x^2-36x+72$. Find the solution set.

If $2i$ is a root, then $-2i$ is a root also.

Factors are: $(x-2i)(x+2i)$

$$x^2+\cancel{2ix}-\cancel{2ix}-4i^2$$
$$x^2-4(-1)$$
$$x^2+4$$

You are not responsible for this

Recall: The complex conjugate of $3-2i$ is $3+2i$

or

$\left.\begin{matrix}-5-2i\\-5+2i\end{matrix}\right\}$ complex conjugates

Then you would divide:

$$x^2+4\,\overline{\big)\,x^4-9x^3+22x^2-36x+72}$$

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Example 9: $3-i$ is a root of $x^3-10x^2+34x-40=0$. Solve the equation.

If $3-i$ is a root, then $3+i$ is also a root.

Example 10: Factor the polynomial over the a) rational numbers; b) real numbers; c) complex numbers.

x^4+4x^2-45

Example 11: Find a polynomial of degree 3 that has zeros 3 and $-2i$ and has $f(-1)=40$.